Differentiation Collated Past Papers – Parametric Equations

2022 Question 1d.

(d)	$\frac{dx}{dt}=3$ $\frac{dy}{dt}=3-\frac{3}{3t-1}$ $=\frac{3(3t-1)-3}{3t-1}$ $=\frac{9t-6}{3t-1}$ $\frac{dy}{dx}=\frac{9t-6}{3t-1}\times\frac{1}{3}$ $=\frac{3t-2}{3t-1}$ $\frac{dy}{dx}=\frac{1}{2}$ $\frac{3t-2}{3t-1}=\frac{1}{2}$ $6t-4=3t-1$ $3t=3$ $t=1$ $x=5 \quad y=3-\ln 2$ or 2.307	Correct expression for $\frac{dy}{dx}$.	Correct solution with correct $\frac{dy}{dx}$.	

2021 Question 1d.

(d)	$x=t^2+3t$ $\frac{dx}{dt}=2t+3$ $y=t^2\ln(2t-3)$ $\frac{dy}{dt}=2t\ln(2t-3)+\frac{2t^2}{2t-3}$ $\frac{dy}{dx}=\frac{2t\ln(2t-3)+\frac{2t^2}{2t-3}}{2t+3}$ At $(10,0)$: $t^2+3t=10$ $t^2+3t-10=0$ $(t+5)(t-2)=0$ $t=-5$ or $t=2$ Since $t>\frac{3}{2}$, $t=2$ $\frac{dy}{dx}=\frac{4\ln(1)+8}{7}$ $\frac{dy}{dx}=\frac{8}{7}$	$\frac{dy}{dt}$ correct.	$\frac{dy}{dt}$ correct And $t^2+3t=10$ solved to find $t=-5$ or $t=2$	T1: Correct solution with correct $\frac{dy}{dx}$.

2020 Question 2e.

(e)	$\frac{dx}{dt}=\frac{1}{t}$ $\quad \frac{dy}{dt}=18t^2$ $\frac{dy}{dx}=18t^3$ $\frac{d^2y}{dx^2}=\frac{d}{dt}\left(\frac{dy}{dx}\right).\frac{dt}{dx}$ $=54t^2\times t$ $=54t^3$ $54t^3=2$ $t^3=\frac{1}{27}$ $t=\frac{1}{3}$ $x=\ln\left(\frac{1}{3}\right)$ $y=6\left(\frac{1}{3}\right)^3$ $=\frac{2}{9}$ P is $\left(\ln\left(\frac{1}{3}\right),\frac{2}{9}\right)$	Correct expression for $\frac{dy}{dx}$.	Correct expression for $\frac{d^2y}{dx^2}$.	Correct solution with correct derivatives. Accept (–1.1, 0.22).

2019 Question 2c.

(c)	$x = \dfrac{1}{(5-t)^2} = (5-t)^{-2}$ $\dfrac{dx}{dt} = -2(5-t)^{-3} \times -1$ $= \dfrac{2}{(5-t)^3}$ $\dfrac{dy}{dt} = 5 - 2t$ $\dfrac{dy}{dx} = \dfrac{(5-2t)(5-t)^3}{2}$ At $t = 2$, $\dfrac{dy}{dx} = \dfrac{1 \times 3^3}{2} = 13.5$	Correct expression for $\dfrac{dx}{dt}$.	Correct solution with correct derivatives shown.	

2019 Question 2e.

(e)	LHS $y = e^{\sin 2x}$ $\dfrac{dy}{dx} = e^{\sin 2x} \times 2\cos 2x$ $\dfrac{d^2y}{dx^2} = e^{\sin 2x} \times (-4\sin 2x) + e^{\sin 2x} \times (2\cos 2x)^2$ $u = \sin 2x$ $\dfrac{du}{dx} = 2\cos 2x$ $\dfrac{d^2u}{dx^2} = -4\sin 2x$ $y = e^u$ $\dfrac{dy}{du} = e^u$ $\dfrac{d^2y}{du^2} = e^u$ RHS $\dfrac{d^2y}{du^2} \times \left(\dfrac{du}{dx}\right)^2 + \dfrac{dy}{du} \times \dfrac{d^2u}{dx^2}$ $= e^u \times (2\cos 2x)^2 + e^u \times (-4\sin 2x)$ $= e^{\sin x} \times (2\cos 2x)^2 + e^{\sin x} \times (-4\sin 2x)$ Therefore LHS = RHS as required. $\dfrac{d^2y}{dx^2} = 4e^{\sin 2x}(\cos^2 2x - \sin 2x)$	Correct expression for $\dfrac{dy}{dx}$ or $\dfrac{du}{dx}$.	Correct expressions for $\dfrac{d^2y}{dx^2}$ in any equivalent form. Or correct RHS.	Complete proof. Accept in terms of x, y, and u.

2018 Question 1e.

(e)	$\frac{dx}{dt} = 3t^2 \quad \frac{dy}{dt} = 2t$ $\frac{dy}{dx} = \frac{dy}{dt} \times \frac{dt}{dx} = 2t \cdot \frac{1}{3t^2} = \frac{2}{3t}$ $\frac{d^2y}{dx^2} = \frac{d\left(\frac{dy}{dx}\right)}{dt} \times \frac{dt}{dx}$ $= \frac{-2}{3t^2} \times \frac{1}{3t^2} = \frac{-2}{9t^4}$ $\frac{\frac{d^2y}{dx^2}}{\left(\frac{dy}{dx}\right)^4} = \frac{\frac{-2}{9t^4}}{\left(\frac{2}{3t}\right)^4}$ $= \frac{-2}{9t^4} \times \frac{81t^4}{16}$ $= \frac{-9}{8}$ or -1.125	Correct $\frac{dy}{dx}$	Correct $\frac{d^2y}{dx^2}$	Correct solution with correct derivatives.

2018 Question 3b.

(b)	$\frac{dx}{dt} = 10e^{2t}$ $\frac{dy}{dt} = 10e^{5t}$ $\frac{dy}{dx} = \frac{10e^{5t}}{10e^{2t}} = \frac{e^{5t}}{e^{2t}}$ $t = 0 \Rightarrow \frac{dy}{dx} = 1$	Correct solution with correct derivatives.		

2017 Question 1d.

(d)	$\frac{dx}{dt} = \frac{1}{2}(t+1)^{\frac{-1}{2}} = \frac{1}{2\sqrt{t+1}}$ $\frac{dy}{dt} = 2\cos 2t$ $\frac{dy}{dx} = 2\cos 2t . 2\sqrt{t+1}$ $= 4\cos 2t . \sqrt{t+1}$ At $t = 0$ $\quad \frac{dy}{dx} = 4\cos 0 \times \sqrt{1}$ $= 4$	$\frac{dx}{dt}$ or $\frac{dy}{dt}$ correct.	Correct solution with correct derivatives.	

2016 Question 1c.

(c)	$\frac{dx}{dt} = -4\sin 2t$ $\frac{dy}{dt} = 2\tan t \sec^2 t$ $\frac{dy}{dx} = \frac{2\tan t \sec^2 t}{-4\sin 2t}$ $= \frac{2\tan t}{-4\sin 2t \cos^2 t}$ At $t = \frac{\pi}{4}$, $\frac{dy}{dx} = \frac{2}{-4\times\left(\frac{1}{\sqrt{2}}\right)^2}$ $= \frac{2}{-2} = -1$	Correct $\frac{dx}{dt}$ or $\frac{dy}{dt}$	Correct solution with correct derivatives.	

2015 Question 3c.

(c)	$\frac{dx}{dt} = -3\sin t$ $\frac{dy}{dt} = 3\cos 3t$ $\frac{dy}{dx} = \frac{3\cos 3t}{-3\sin t} = \frac{-\cos 3t}{\sin t}$ At $t = \frac{\pi}{4}$, $\frac{dy}{dx} = \frac{-\cos\left(\frac{3\pi}{4}\right)}{\sin\left(\frac{\pi}{4}\right)} = 1$ $\therefore$ Gradient of normal $= -1$	Correct $\frac{dx}{dt}$ and $\frac{dy}{dt}$	Correct solution with correct derivatives.	

2014 Question 1c.

(c)	$x = 2\sin t \quad y = \cos 2t$ $\frac{dx}{dt} = 2\cos t \quad \frac{dy}{dt} = -2\sin 2t$ $\frac{dy}{dx} = \frac{-2\sin 2t}{2\cos t}$ $= \frac{-2\times 2\sin t\cos t}{2\cos t}$ $= -2\sin t$	Correct expressions for $\frac{dx}{dt}$ and $\frac{dy}{dt}$.	A correct solution.	

2013 Question 1d.

(d)	$\frac{dx}{dt} = 5\cos t \qquad \frac{dy}{dt} = 3\sec^2 t = \frac{3}{\cos^2 t}$ $\frac{dy}{dx} = \frac{3}{5\cos^3 t}$ At $t = \frac{\pi}{3}$, $\frac{dy}{dx} = \frac{24}{5} (= 4.8)$ $\therefore$ gradient of normal $= \frac{-5}{24} (= -0.2083)$	Correct $\frac{dx}{dt}$ and $\frac{dy}{dt}$.	Correct solution including all correct derivatives.	

2013 Question 3d.

(d)	For the curve, $\frac{dy}{dx} = \frac{3t^2 - 3}{2t - 1}$ Normal parallel to the y-axis means tangent parallel to the x-axis. $\Rightarrow \frac{dy}{dx} = 0$ $3t^2 = 3$ $t = \pm 1$ $t = 1 \Rightarrow$ point $(0,-2)$ $t = -1 \Rightarrow$ point $(2,2)$	Correct expression for $\frac{dy}{dx}$	Correct solution with correct derivative.	