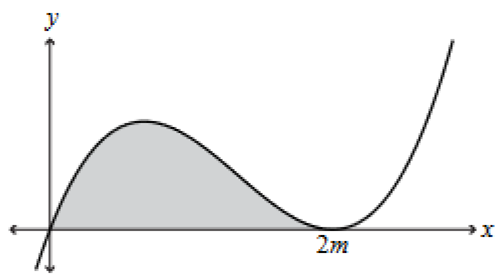
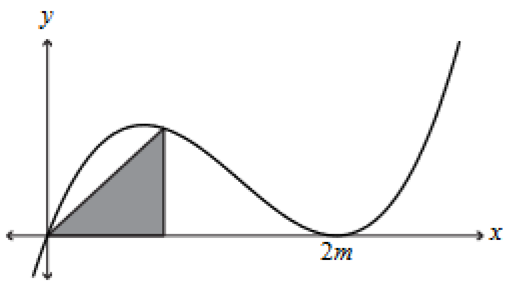
Differentiation Collated Past Papers - Extrema

2023 Question 1e.

(e) The graph of $y = x(x-2m)^2$, where $m > 0$, is shown. The total shaded area between the curve and the x-axis from $x = 0$ to $x = 2m$ is given by $A = \frac{4m^4}{3}$.



A right-angled triangle is now constructed with one vertex at (0,0) and another on the curve $y = x(x-2m)^2$, as shown below.



Show that the maximum area of such a triangle is $\frac{3}{8}$ of the total shaded area.

You must use calculus and show any derivatives that you need to find when solving this problem.

You do not have to prove that the area you have found is a maximum.

2023 Question 2d.

(d) Find the x-value(s) of any points of inflection on the graph of the function $f(x) = 3x^2\ln(x)$.

You can assume that your point(s) found are actually point(s) of inflection.

You must use calculus and show any derivatives that you need to find when solving this problem.

2023 Question 3d.

(d) Find the co-ordinates of any stationary points on the graph of the function $f(x) = \frac{1}{x} - \frac{2}{x^3}$, identifying their nature.

You must use calculus and show any derivatives that you need to find when solving this problem.

2023 Question 3e.

(e) A power line hangs between two poles.

The equation of the curve $y = f(x)$ that models the shape of the power line can be found by solving the differential equation:

$$a\frac{\mathrm{d}^2y}{\mathrm{d}x^2} = \sqrt{1+\left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)^2}$$

Use differentiation to verify that the function $y = \frac{a}{2}\left(\mathrm{e}^{\frac{x}{a}} + \mathrm{e}^{-\frac{x}{a}}\right)$

satisfies the above differential equation, where a is a positive constant.

2022 Question 1b.

(b) Find the x-value(s) of any stationary points on the graph of the function $f(x) = \frac{x^2+1}{x}$.

You must use calculus and show any derivatives that you need to find when solving this problem.

2022 Question 1e.

(e) If p is a positive real constant, prove that $y = \mathrm{e}^{px^2}$ does not have any points of inflection.

You must use calculus and show any derivatives that you need to find when solving this problem.

2022 Question 2c.

(c) An object is travelling in a straight line. Its displacement, in metres, is given by the formula:

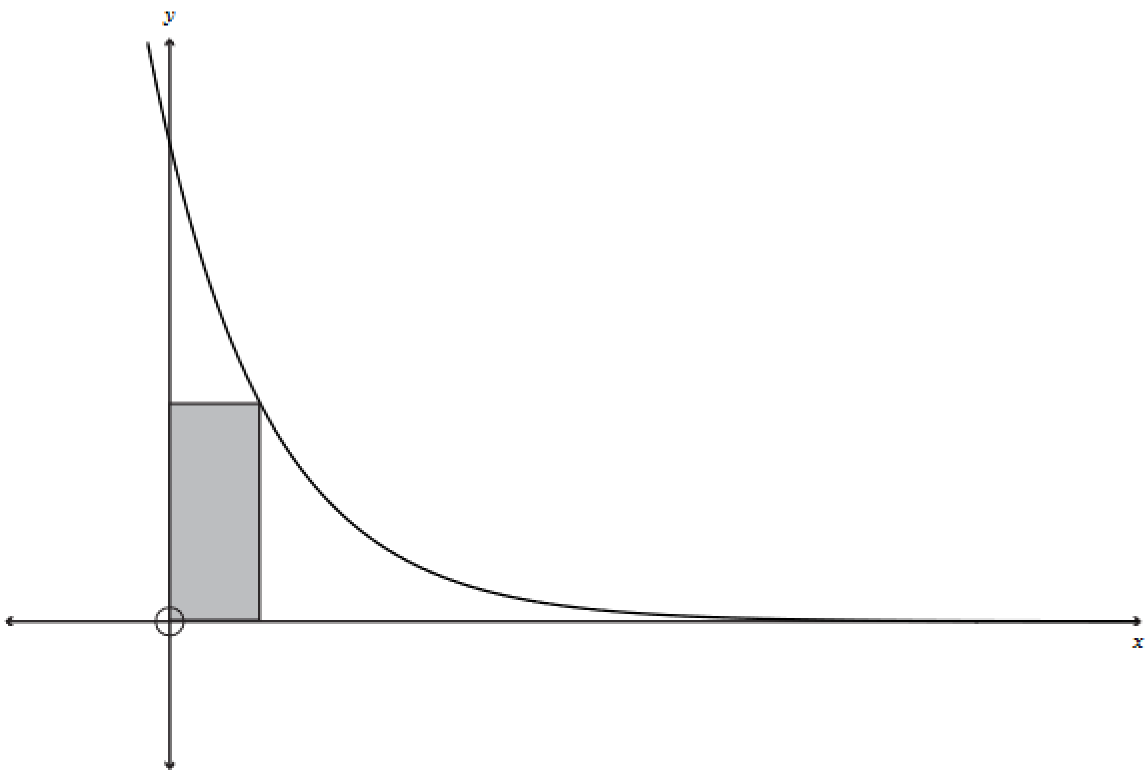
$$d(t) = \frac{t^2-6}{2t^3} \text{ where } t > 0,\ t \text{ is time in seconds.}$$

Find the time(s) when the object is stationary.

You must use calculus and show any derivatives that you need to find when solving this problem.

2022 Question 2d.

(d) A rectangle has one vertex at (0,0) and the opposite vertex on the curve $y = 6e^{1-0.5x}$, where $x > 0$, as shown on the graph below.



Find the maximum possible area of the rectangle.

You must use calculus and show any derivatives that you need to find when solving this problem.

You do not have to prove that the area you have found is a maximum.

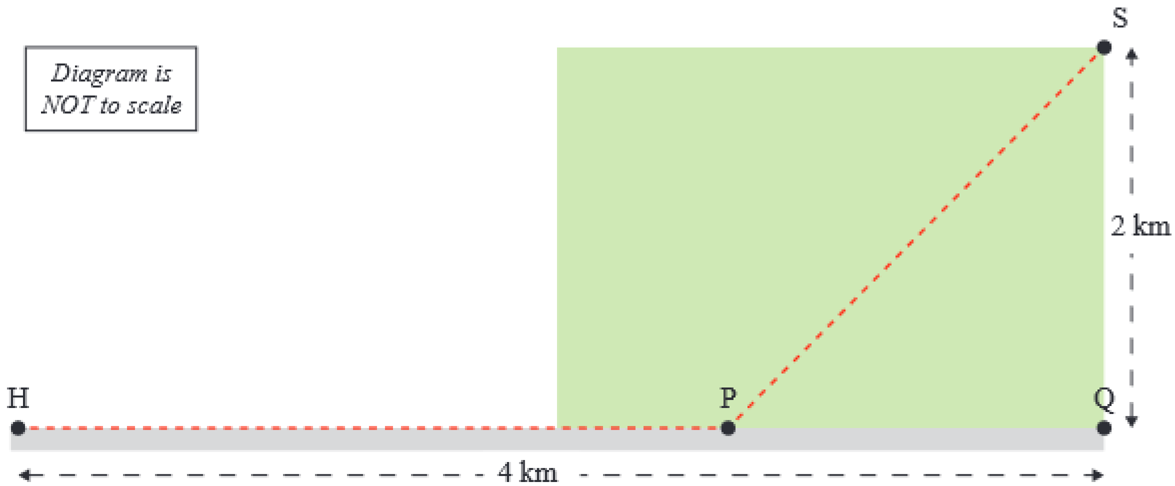
2022 Question 3d.

(d) Find the x-value(s) of any stationary point(s) on the graph of the function $y = 9x - 2 + \dfrac{3}{3x-1}$ and determine their nature.

You must use calculus and show any derivatives that you need to find when solving this problem.

2022 Question 3e.

(e) Megan cycles from her home, H, to school, S, each day.



She rides along a path from her home to point P at a constant speed of 10 kilometres per hour.

At point P, Megan cuts across a park, heading directly to school. When cycling across the park, Megan can only cycle at 6 kilometres per hour.

At what distance from her home should she choose to cut across the park in order to make her travelling time a minimum?

You must use calculus and show any derivatives that you need to find when solving this problem.

2021 Question 1c.

(c) A curve has the equation $y = (2x + 3)e^{x^2}$.

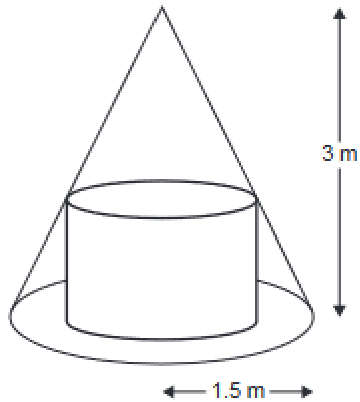
Find the x-coordinate(s) of any stationary point(s) on the curve.

You must use calculus and show any derivatives that you need to find when solving this problem.

2021 Question 1e.

(e) A cone has a height of 3 m and a radius of 1.5 m.

A cylinder is inscribed in the cone, as shown in the diagram below.



The base of the cylinder has the same centre as the base of the cone.

Prove that the maximum volume of the cylinder is π m^3.

You must use calculus and show any derivatives that you need to find when solving this problem.

2021 Question 2b.

(b) A curve has the equation $y = \dfrac{x^2}{x+1}$.

Find the x-coordinate(s) of any stationary point(s) on the curve.

You must use calculus and show any derivatives that you need to find when solving this problem.

2021 Question 3c.

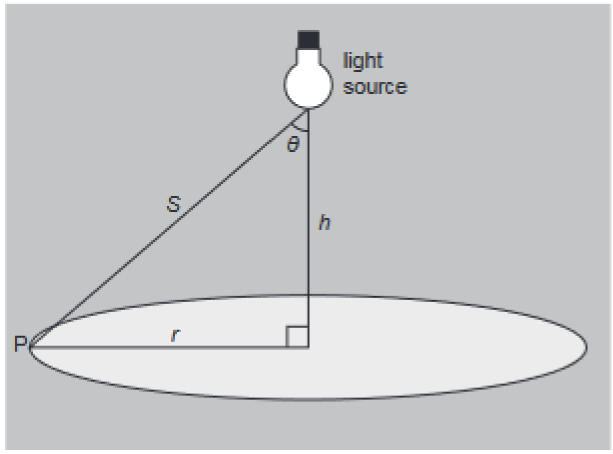
(c) For what values of x is the function $y = \dfrac{x}{x^2+4}$ increasing?

You must use calculus and show any derivatives that you need to find when solving this problem.

2021 Question 3e.

(e) A lamp is suspended above the centre of a round table of radius r.

The height, h, of the lamp above the table is adjustable.



Point P is on the edge of the table.

At point P the illumination I is directly proportional to the cosine of angle θ in the above diagram, and inversely proportional to the square of the distance, S, to the lamp.

i.e. $I = \dfrac{k\cos\theta}{S^2}$, where k is a constant.

Prove that the edge of the table will have maximum illumination when $h = \dfrac{r}{\sqrt{2}}$.

You do not need to prove that your solution gives the maximum value.

You must use calculus and show any derivatives that you need to find when solving this problem.

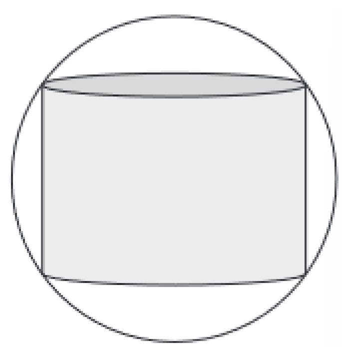
2020 Question 1c.

(c) Find the value of x for which the graph of the function $y = \dfrac{x}{1+\ln x}$ has a stationary point.

You must use calculus and show any derivatives that you need to find when solving this problem.

2020 Question 1e.

(e) A cylinder of height h and radius r is inscribed, as shown to the right, inside a sphere of radius 20 cm.



Find the maximum possible volume of the cylinder.

You must use calculus and show any derivatives that you need to find when solving this problem.

You do not need to prove that the volume you have found is a maximum.

2020 Question 2c.

c) Find the x-coordinates of any stationary points on the graph of the function

$$f(x) = (2x-3)e^{x^2+k}$$

You must use calculus and show any derivatives that you need to find when solving this problem.

2020 Question 3d.

d) The graph of the function $y = \frac{1}{x-3} + x,\ x \neq 3$, has two stationary points.

Find the x-coordinates of the stationary points, and determine whether they are local maxima or local minima.

You must use calculus and show any derivatives that you need to find when solving this problem.

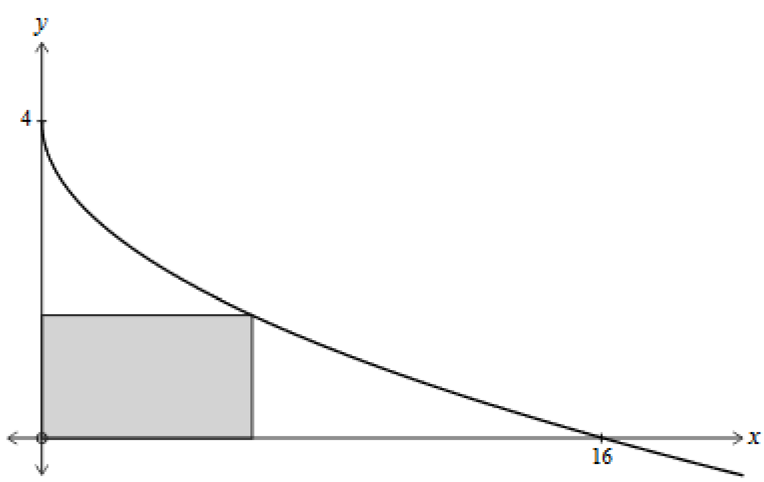
2019 Question 1d.

d) For what value(s) of x is the function $y = x^3e^x$ decreasing?

You must use calculus and show any derivatives that you need to find when solving this problem.

2019 Question 3c.

c) A rectangle has one vertex at (0,0), and the opposite vertex on the curve $y = 4 - \sqrt{x}$, where $0 < x < 16$, as shown on the graph below.



Find the maximum possible area of the rectangle.

You must use calculus and show any derivatives that you need to find when solving this problem.

You do not need to prove that the area you have found is a maximum.

2019 Question 3d.

(d) The velocity of an object is modelled by the function

$$v = 2e^{t} + 8e^{-t}, \text{ for } t \geq 0$$

where v is the velocity of the object, in m s^{-1}
and t is the time in seconds since the start of the object's motion.

Find the time when the acceleration of the object is 0.

You must use calculus and show any derivatives that you need to find when solving this problem.

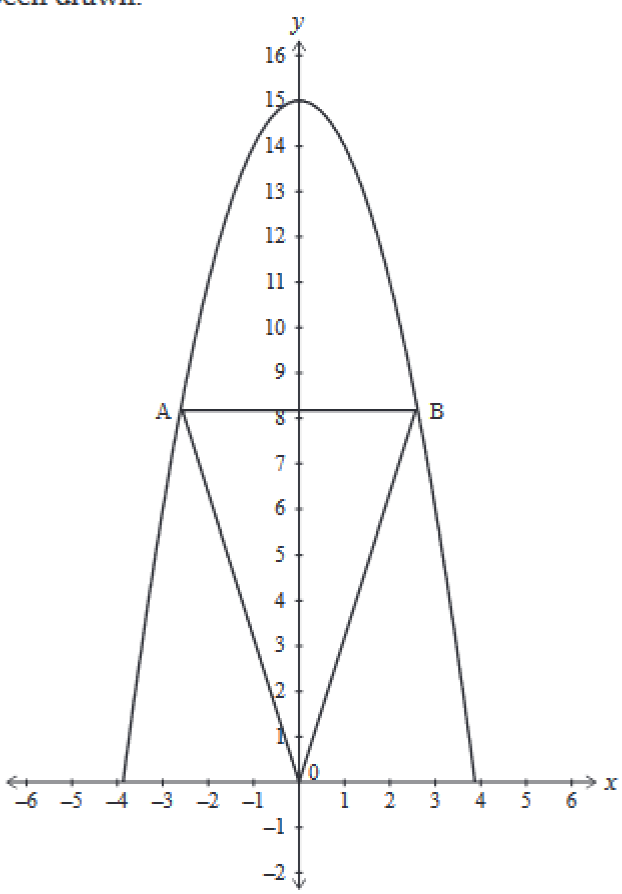
2018 Question 2d.

(d) If $y = e^{x}(2x^2 - x - 1)$, find the value(s) of x for which $\frac{dy}{dx} = 0$.

You must use calculus and show any derivatives that you need to find when solving this problem.

2018 Question 3c.

(c) The diagram below shows the graph of the function $y = 15 - x^2$, inside which an isosceles triangle OAB has been drawn.



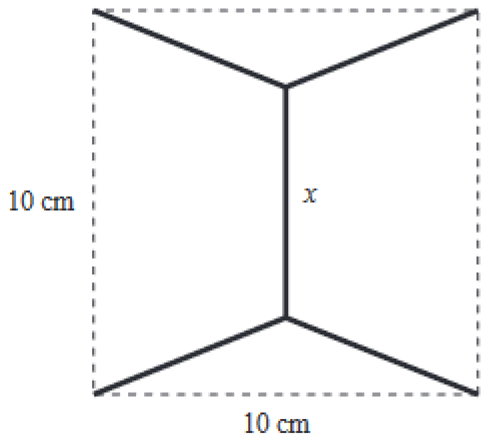
Find the maximum possible area, A, of the triangle.

You may assume that your answer is a maximum.

You must use calculus and show any derivatives that you need to find when solving this problem.

2018 Question 3e.

e)



The above shape is made from wire. It has both vertical and horizontal lines of symmetry.

The ends of the shape are at the vertices of a square with a side length of 10 cm, as shown in the diagram above.

The length of the piece of wire through the centre of the shape is x cm.

Find the value(s) of x that enables the shape to be made with the minimum length of wire.

You do not need to prove that the length is a minimum.

You must use calculus and show any derivatives that you need to find when solving this problem.

2017 Question 1e.

e) Find the values of a and b such that the curve $y = \dfrac{ax-b}{x^2-1}$ has a turning point at (3,1).

You must use calculus and show any derivatives that you need to find when solving this problem.

2017 Question 2b.

b) The percentage of seeds germinating depends on the amount of water applied to the seedbed that the seeds are sown in, and may be modelled by the function:

$$P(w) = 96\ln(w + 1.25) - 16w - 12$$

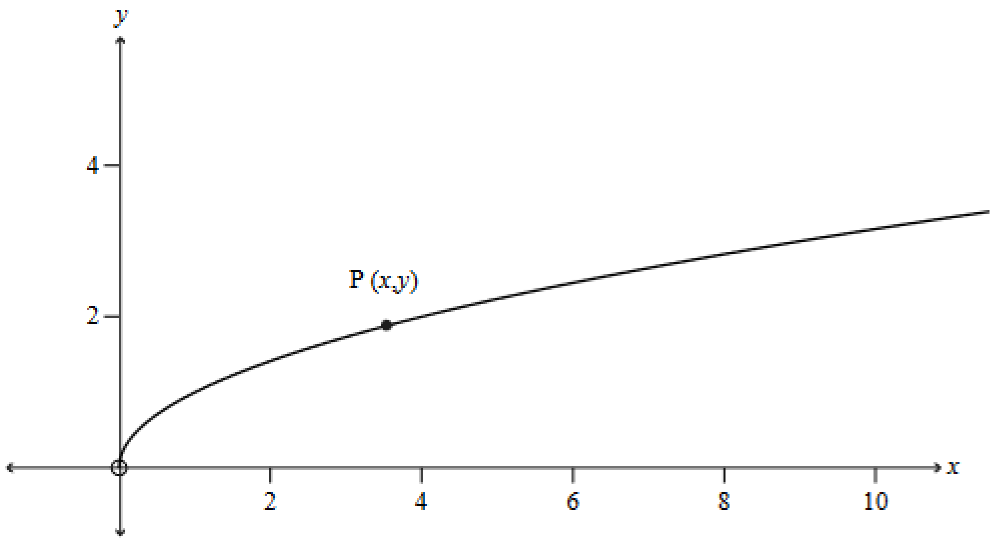
where P is the percentage of seeds that germinate and
w is the daily amount of water applied (litres per square metre of seedbed), with $0 \le w \le 15$.

Find the amount of water that should be applied daily to maximise the percentage of seeds germinating.

You must use calculus and show any derivatives that you need to find when solving this problem.

2017 Question 2d.

(d) Find the coordinates of the point P (x,y) on the curve $y=\sqrt{x}$ that is closest to the point $(4,0)$.

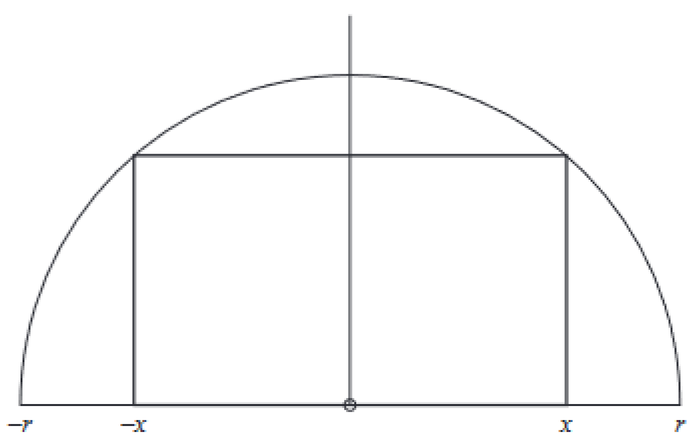


You do not need to prove that your solution is the minimum value.

You must use calculus and show any derivatives that you need to find when solving this problem.

2017 Question 2e.

(e) A rectangle is inscribed in a semi-circle of radius r, as shown below.



Show that the maximum possible area of such a rectangle occurs when $x=\dfrac{r}{\sqrt{2}}$.

You do not need to prove that your solution gives the maximum area.

You must use calculus and show any derivatives that you need to find when solving this problem.

2016 Question 1e.

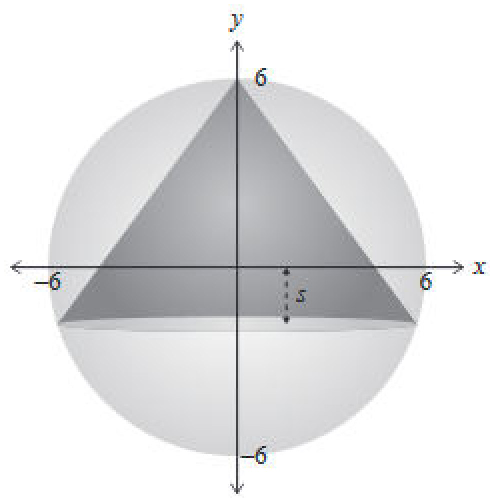
e) A curve is defined by the function $f(x) = e^{-(x-k)^2}$.

Find, in terms of k, the x-coordinate(s) for which $f''(x) = 0$.

You must use calculus and show any derivatives that you need to find when solving this problem.

2016 Question 2e.

e) A cone of height h and radius r is inscribed, as shown, inside a sphere of radius 6 cm.



The base of the cone is s cm below the x-axis.

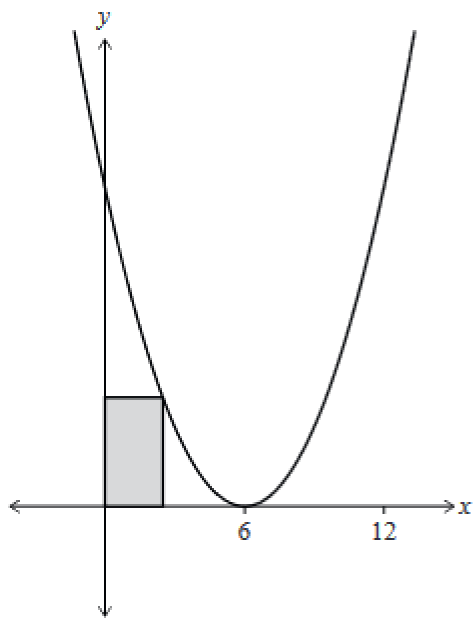
Find the value of s which maximises the volume of the cone.

You must use calculus and show any derivatives that you need to find when solving this problem.

You do not need to prove that the volume you have found is a maximum.

2016 Question 3c.

(c) A rectangle has one vertex at (0,0) and the opposite vertex on the curve $y = (x - 6)^2$, where $0 < x < 6$, as shown on the graph below.



Find the maximum possible area of the rectangle.

You must use calculus and show any derivatives that you need to find when solving this problem.

You do not need to prove that the area you have found is a maximum.

2015 Question 1c.

(c) Find the values of x for which the function $f(x) = 8x - 3 + \frac{2}{x+1}$ is increasing.

You must use calculus and show any derivatives that you need to find when solving this problem.

2015 Question 3b.

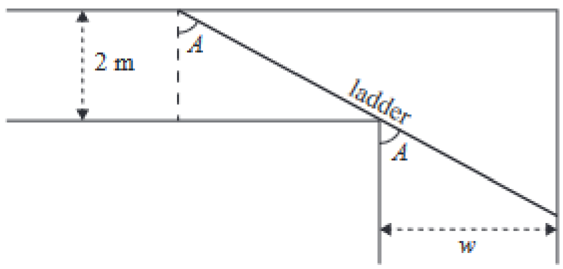
(b) If $f(x) = \frac{x}{e^{3x}}$, find the value(s) of x such that $f'(x) = 0$.

You must use calculus and show any derivatives that you need to find when solving this problem.

2015 Question 3e.

e) A corridor is 2 m wide.

At the end it turns 90° into another corridor.

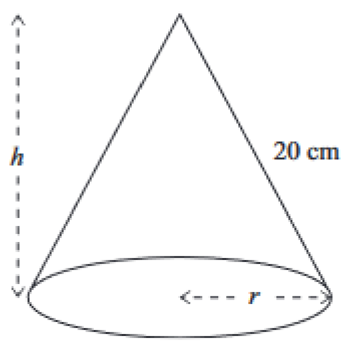


What is the minimum width, w, of the second corridor if a ladder of length 5 m can be carried horizontally around the corner?

You must use calculus and show any derivatives that you need to find when solving this problem.

2014 Question 1e.

e) What is the maximum volume of a cone if the slant length of the cone is 20 cm?



You do not need to prove that the volume you have found is a maximum.

Show any derivatives that you need to find when solving this problem.

2014 Question 2d.

d) The hourly cost of running an aeroplane depends on the speed at which it flies.

For a particular aeroplane this is given by the equation

$$C = 4v + \frac{1\,000\,000}{v}, \quad 200 \le v \le 800$$

where C is the hourly cost of running the aeroplane, in dollars per hour and v is the airspeed of the aeroplane, in kilometres per hour.

Find the minimum hourly cost at which this aeroplane can be flown.

Show any derivatives that you need to find when solving this problem.

2014 Question 3b.

(b) Find the value(s) of x for which the graph of the function $y = x + \frac{32}{x^2}$ has stationary points.

Show any derivatives that you need to find when solving this problem.

2014 Question 3c.

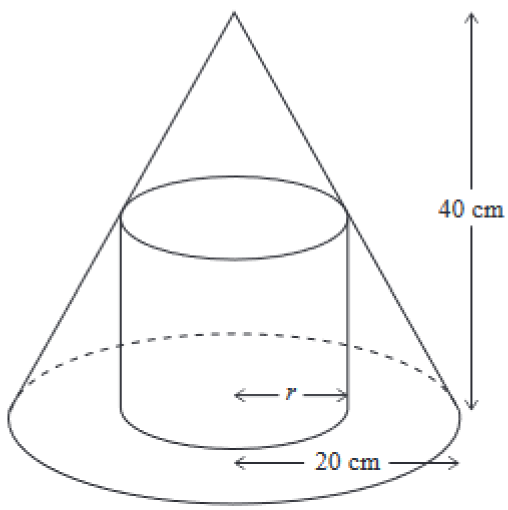
(c) For what values of x is the function $f(x) = 5x - x\ln x$ increasing?

Show any derivatives that you need to find when solving this problem.

2014 Question 3e.

(e) A cone has a radius of 20 cm and a height of 40 cm.

A cylinder fits inside the cone, as shown below.



What must the radius of the cylinder be to give the cylinder the maximum volume?

You do not need to prove that the volume you have found is a maximum.

Show any derivatives that you need to find when solving this problem.

2013 Question 1c.

(c) Find the x values of any points of inflection on the graph of the function $y = e^{(6-x^2)}$.

Show any derivatives that you need to find when solving this problem.

2013 Question 1e.

(e) A closed cylindrical tank is to have a surface area of 20 m^2.

Find the radius the tank needs to have so that the volume it can hold is as large as possible.

You do not have to prove that your solution gives the maximum volume.

Show any derivatives that you need to find when solving this problem.

2013 Question 2c.

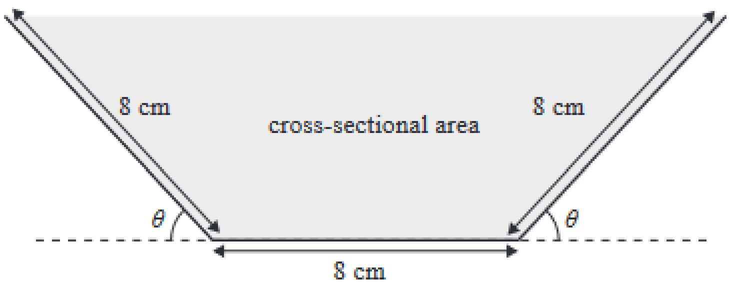
(c) For what value of k does the function $f(x) = x - e^x - \frac{k}{x}$ have a stationary point at $x = -1$?

Show any derivatives that you need to find when solving this problem.

2013 Question 2e.

(e) A copper sheet of width 24 cm is folded, as shown, to make spouting.

Cross-section:



Find angle θ which gives the maximum cross-sectional area.

You do not need to prove that you have found a maximum.

Show any derivatives that you need to find when solving this problem.

2013 Question 3b.

(b) For the function $f(x) = x + \frac{16}{x-2}$, find the x-values of any stationary points.

You must use calculus and clearly show your working, including any derivatives you need to find when solving this problem.

2013 Question 3c.

(c) Find the value of x that gives the maximum value of the function

$$f(x) = 50x - 30x \ln 2x$$

You do not need to prove that your value of x gives a maximum.

You must use calculus and clearly show your working, including any derivatives you need to find when solving this problem.