Differentiation Collated Past Papers - Rates

2023 Question 1b.

(b) Find the rate of change of the function $f(t) = t^2 e^{2t}$ when $t = 1.5$.

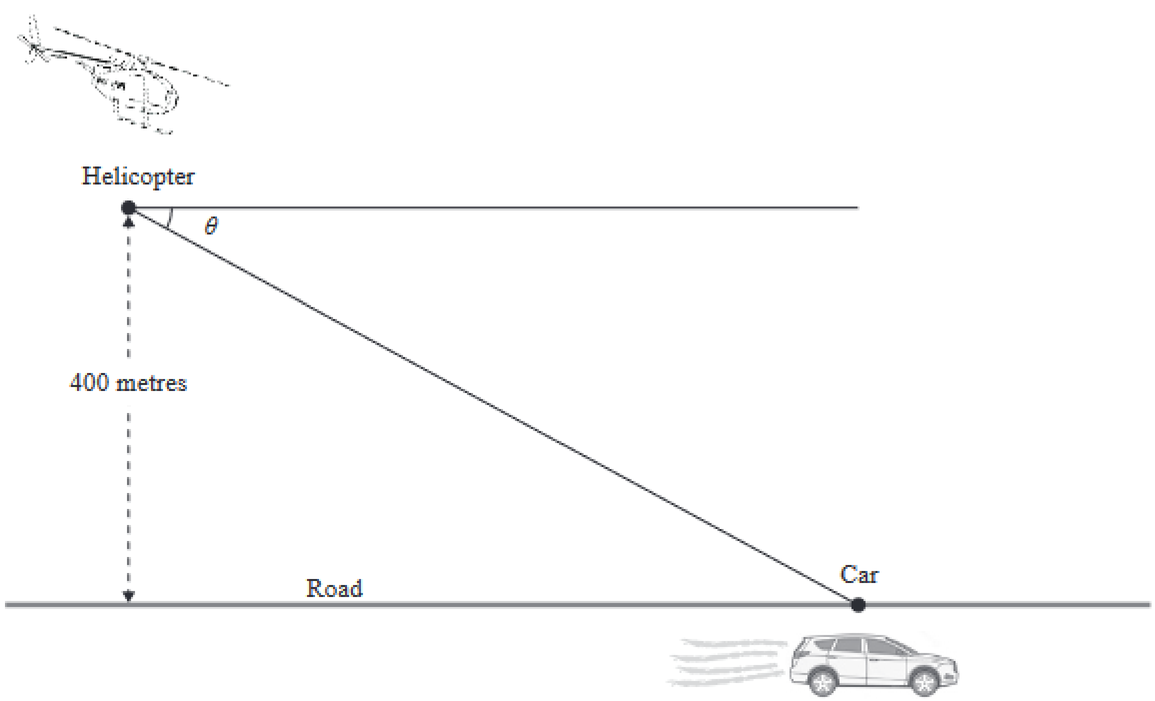
You must use calculus and show any derivatives that you need to find when solving this problem.

2023 Question 2e.

(e) A police helicopter is flying above a straight horizontal section of motorway chasing a speeding car.

The helicopter is flying at a constant speed of 72 m s^{-1} and at a constant height of 400 metres above the ground. The helicopter is attempting to catch up with the car.

When the direct distance from the helicopter to the car is 2500 metres, the angle of depression, θ, between the horizontal and the line of sight from the helicopter to the car is increasing at a rate of 0.002 rad s^{-1}.



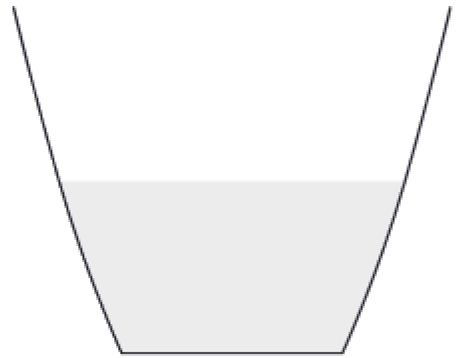
Adapted from: https://animalia-life.club/qa/pictures/police-helicopter-drawing, https://www.freepik.com/premium-vector/car-sedan-suv-drawing-outlines-converted-objects_17981778.htm

Calculate the speed of the **car** at this instant.

You must use calculus and show any derivatives that you need to find when solving this problem.

2022 Question 3c.

(c) The diagram below shows the cross-section of a bowl containing water.



When the height of the water level in the bowl is h cm, the volume, V cm^3, of water in the bowl is given by $V = \pi\left(\frac{3}{2}h^2 + 3h\right)$.

Water is poured into the bowl at a constant rate of 20 cm^3 s^{-1}.

Find the rate, in cm s^{-1}, at which the height of the water level is increasing when the height of the water level is 3 cm.

You must use calculus and show any derivatives that you need to find when solving this problem.

2021 Question 2d.

(d) The volume of a spherical balloon is increasing at a constant rate of 60 cm^3 per second.

Find the rate of increase of the radius when the radius is 15 cm.

You must use calculus and show any derivatives that you need to find when solving this problem.

2020 Question 2b.

(b) The value of a car is modelled by the formula

$V = 17\,000\,e^{-0.25t} + 2\,000\,e^{-0.5t} + 500$ for $0 \le t \le 20$

where V is the value of the car in dollars ($), and t is the age of the car in years.

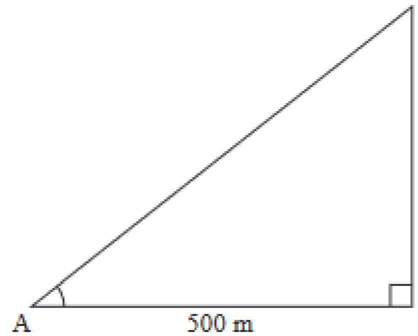
Calculate the rate at which the value of the car is changing when it is 8 years old.

You must use calculus and show any derivatives that you need to find when solving this problem.

2020 Question 2d.

(d) A rocket is fired vertically upwards. Its height above the launch point is given by the formula $h(t) = 4.8t^2$, where h is the height in metres, and t is the time in seconds from firing.

www.airspacemag.com/as-next/milestone-180968351/



An observer at point A is watching the rocket. She is at the same level as the launch point of the rocket, and 500 m from the launch point.

Find the rate at which the angle of elevation at A of the rocket is increasing when the rocket is 480 m above the launch point.

You must use calculus and show any derivatives that you need to find when solving this problem.

2019 Question 1e.

(e) The volume of a sphere is increasing.

At the instant when the sphere's radius is 0.5 m, the surface area of the sphere is increasing at a rate of $0.4\ m^2\ s^{-1}$.

Find the rate at which the volume of the sphere is increasing at this instant.

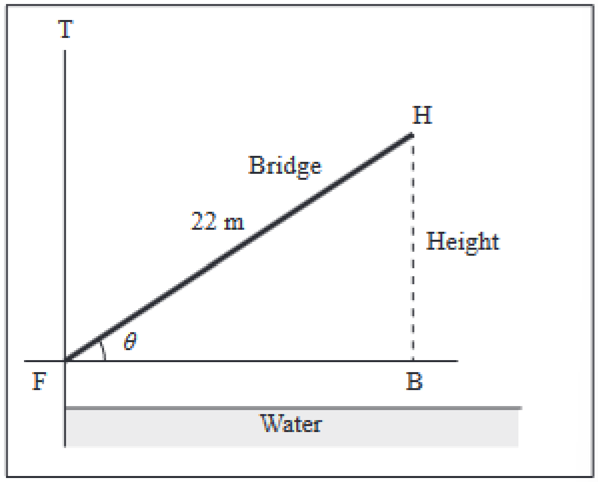
You must use calculus and show any derivatives that you need to find when solving this problem.

2019 Question 2d.

(d) The Wynyard Crossing bridge in Auckland can be raised and lowered to allow tall boats to sail through when open, and pedestrians to walk across when closed. The bridge consists of two arms, each of length 22 metres.

When the bridge is rising, the angle of the bridge arm above the horizontal increases at the rate of 0.01 rad s^{-1}.

www.youtube.com/watch?v=Q4xrCt-uYPE

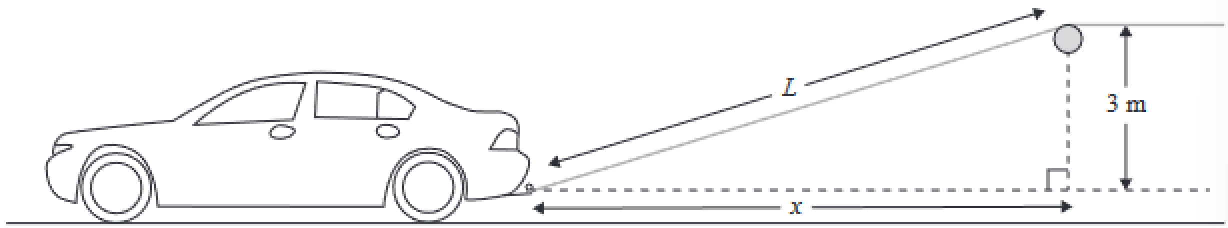


Find the rate at which the height, BH, is increasing when H is 15 metres above the horizontal, FB.

You must use calculus and show any derivatives that you need to find when solving this problem.

2018 Question 1d.

(d)



A car is being pulled along by a rope attached to the tow-bar at the back of the car.

The rope passes through a pulley, the top of which is 3 m further from the ground than the tow-bar.

The pulley is x m horizontally from the tow-bar, as shown in the diagram above.

The rope is being winched in at a speed of 0.6 m s^{-1}.

The wheels of the car remain in contact with the ground.

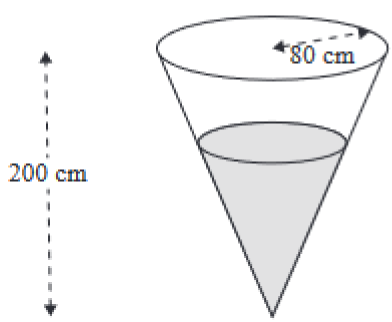
At what speed is the car moving when the length of the rope, L, between the tow-bar and the pulley is 5.4 m?

You must use calculus and show any derivatives that you need to find when solving this problem.

2018 Question 2e.

(e) A water tank is in the shape of an inverted right-circular cone.

The height of the cone is 200 cm and the radius of the cone is 80 cm.



The tank is being filled with water at a rate of 150 cm^3 per second.

At what rate will the surface area of the water in the tank be increasing when the depth of water in the tank is 125 cm?

You must use calculus and show any derivatives that you need to find when solving this problem.

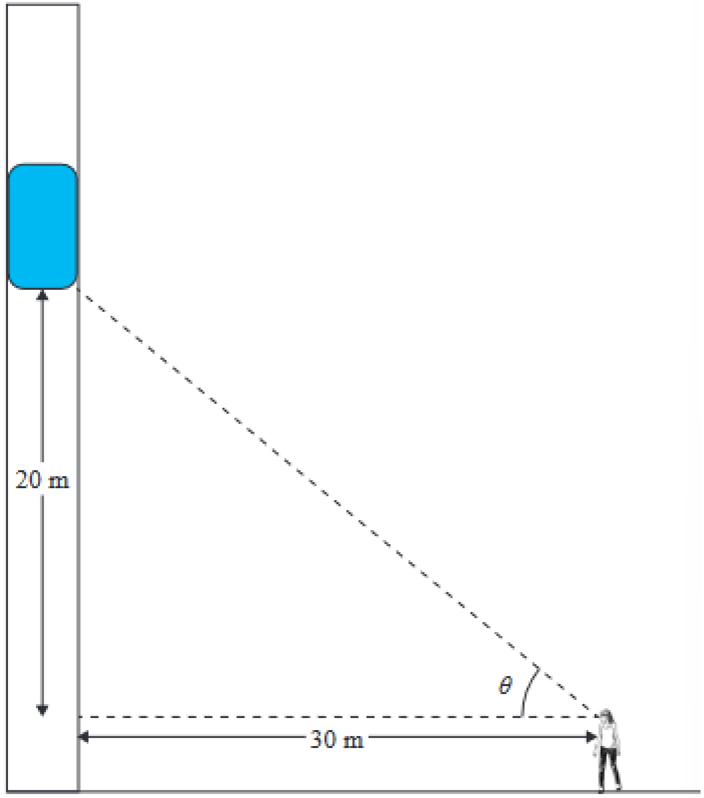
2017 Question 3d.

(d) A building has an external elevator. The elevator is rising at a constant rate of 2 m s^{-1}.

Sarah is stationary, watching the elevator from a point 30 m away from the base of the elevator shaft.

Let the angle of elevation of the elevator floor from Sarah's eye level be θ.

www.alibaba.com/product-detail/Sicher-external-elevator_60136882005.html



Find the rate at which the angle of elevation is increasing when the elevator floor is 20 m above Sarah's eye level.

You must use calculus and show any derivatives that you need to find when solving this problem.

2016 Question 1b.

(b) The height of the tide at a particular beach today is given by the function

$$h(t) = 0.8\sin\left(\frac{4\pi}{25}t + \frac{\pi}{2}\right)$$

where h is the height of water, in metres, relative to the mean sea level and t is the time in hours after midnight.

c2kiwi.blogspot.co.nz/2011/01/christchurch-wedding-stroll-on-beach.html

At what rate was the height of the tide changing at that beach at 9.00 a.m. today?

2016 Question 2d.

(d) A large spherical helium balloon is being inflated at a constant rate of $4800\ \text{cm}^3\ \text{s}^{-1}$.

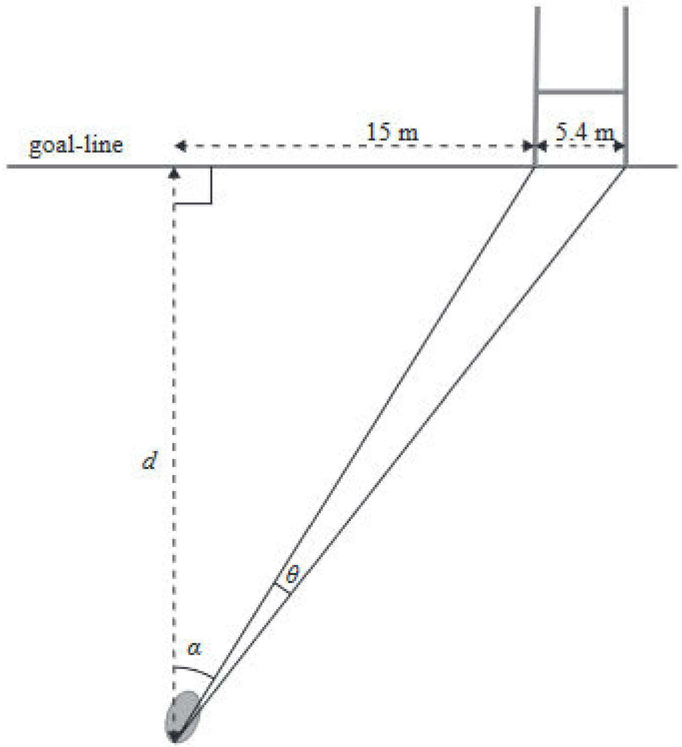
At what rate is the radius of the balloon increasing when the volume of the balloon is $288\,000\pi\ \text{cm}^3$?

You must use calculus and show any derivatives that you need to find when solving this problem.

2016 Question 3e.

(e) In a rugby game, a try is scored 15 m from the left-hand goal-post. The conversion kick is taken at some point on the line perpendicular to the goal-line from the point where the try was scored, as shown in the diagram below.

The ball needs to pass between the goal-posts, which are 5.4 m apart.



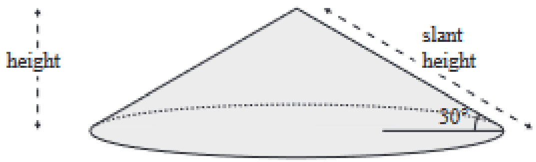
Find the distance d from the goal-line that the conversion kick should be taken from in order to maximise the angle θ between the lines from the ball to the goal-posts.

You must use calculus and show any derivatives that you need to find when solving this problem.

You do not need to prove that the angle you have found is a maximum.

2015 Question 1e.

(e) Salt harvested at the Grassmere Saltworks forms a cone as it falls from a conveyor belt.
The slant of the cone forms an angle of 30° with the horizontal.
The conveyor belt delivers the salt at a rate of 2 m^3 of salt per minute.



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https://bronzblog.files.wordpress.com/2013/07/imgp1182.jpg

Find the rate at which the slant height is increasing when the radius of the cone is 10 m.

You must use calculus and show any derivatives that you need to find when solving this problem.

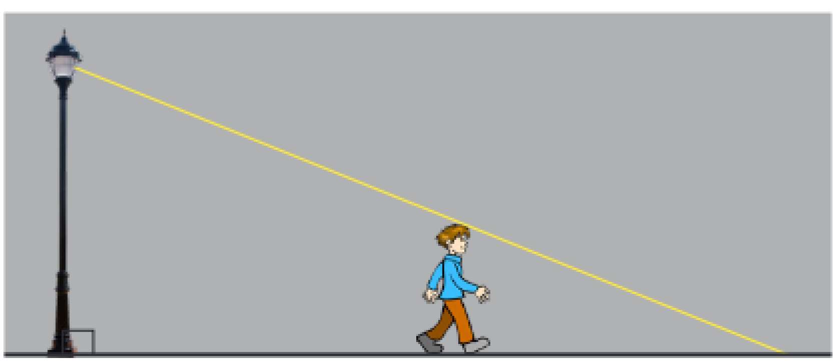
2015 Question 2d.

(d) A street light is 5 m above the ground, which is flat.

A boy, who is 1.5 m tall, is walking away from the point directly below the streetlight at 2 metres per second.

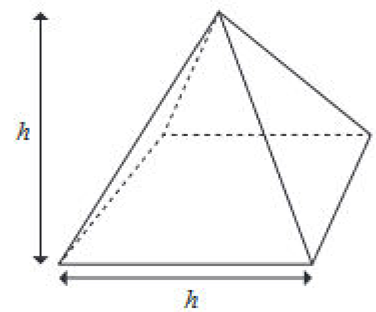
At what rate is the length of his shadow changing when the boy is 8 m away from the point directly under the light?

You must use calculus and show any derivatives that you need to find when solving this problem.



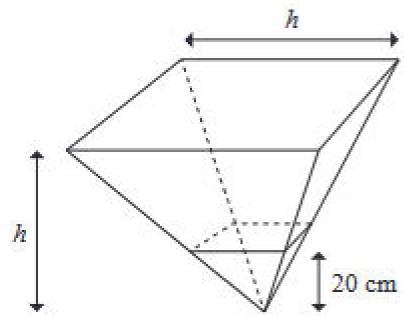
2015 Question 2e.

(e) A water container is constructed in the shape of a square-based pyramid. The height of the pyramid is the same as the length of each side of its base.



A vertical height of 20 cm is then cut off the top of the pyramid, and a new flat top added.

The pyramid is then inverted and water is poured in at a rate of 3000 cm^3 per minute.



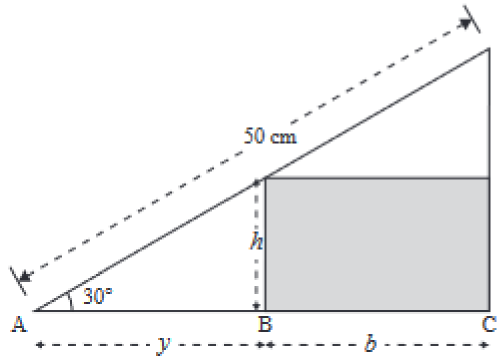
Find the rate at which the surface area of the water is increasing when the depth of the water is 15 cm.

$$\text{Volume of pyramid} = \frac{1}{3} \times \text{base area} \times \text{height}$$

You must use calculus and show any derivatives that you need to find when solving this problem.

2014 Question 2e.

(e) A rectangle is drawn inside a right angled triangle, as shown in the diagram below.



Point B moves along the base of the triangle AC, beginning at point A, at a constant speed of 3 cm s^{-1}.

At what rate is the area of the rectangle changing when point B is 20 cm from point A?

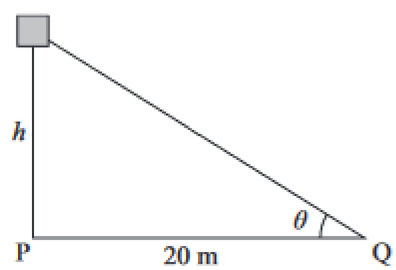
Show any derivatives that you need to find when solving this problem.

2014 Question 3d.

(d) A container is winched up vertically from a point P at a constant rate of 1.5 m s^{-1}.

It is being observed from point Q, which is 20 m horizontally from point P.

θ is the angle of elevation of the container from point Q.



At what rate is the angle of elevation increasing when the object is 20 m above point P?

Show any derivatives that you need to find when solving this problem.

2013 Question 3e.

e) A spherical balloon is being inflated with helium.

The balloon is being inflated in such a way that its volume is increasing at a constant rate of 300 $cm^3\ s^{-1}$.

The material that the balloon is made of is of limited strength, and the balloon will burst when its surface area reaches 7 500 cm^2.

Find the rate at which the surface area of the balloon is increasing when it reaches bursting point.

Show any derivatives that you need to find when solving this problem.