Differentiation Collated Past Papers - Extrema

2023 Question 1e.

(e)	Area of triangle $= \frac{1}{2}xy$ $A = \frac{1}{2}x\left(x(x-2m)^2\right)$ $= \frac{1}{2}x^2(x-2m)^2$ $\frac{dA}{dx} = \frac{1}{2}x^2(2(x-2m)) + (x-2m)^2$ $= x^2(x-2m) + x(x-2m)^2$ $= x(x-2m)\left(x+(x-2m)\right)$ $= x(x-2m)(2x-2m)$ $= 2x(x-2m)(x-m)$ OR $A = \frac{1}{2}x^2(x-2m)^2$ $= \frac{1}{2}x^4 - 2mx^3 + 2m^2x^2$ $\frac{dA}{dx} = 2x^3 - 6mx^2 + 4m^2x$ $= 2x\left(x^2 - 3mx + 2m^2\right)$ $= 2x(x-2m)(x-m)$ $\frac{dA}{dx} = 0 \Rightarrow 2x(x-2m)(x-m) = 0$ $x = 0$ or $x = 2m$ or $x = m$ Since $0 < x < 2m$ the area is a maximum when $x = m$ Maximum area of triangle: $A(m) = \frac{1}{2}m^2\left(m-2m\right)^2$ $= \frac{1}{2}m^4$ This is $\frac{3}{8}$ of the total shaded area since $\frac{3}{8} \times \frac{4m^3}{3} = \frac{1}{2}m^4$	• Correct derivative.	• Correct derivative. AND $x = m$ found.	T1 Maximum area, $A = \frac{1}{2}m^4$ found with correct $\frac{dA}{dx}$. OR Correct solution but with one minor error. T2 Correct solution with correct $\frac{dA}{dx}$ showing the calculation of the correct proportion of total shaded area..

2023 Question 2d.

(d)	$f'(x)=3x^2\cdot\frac{1}{x}+\ln x\cdot(6x)$ $=3x+6x\ln x$ $f''(x)=3+6x\cdot\frac{1}{x}+\ln x(6)$ $=9+6\ln x$ $f''(x)=0\Rightarrow 9+6\ln x=0$ $\ln x=-1.5$ $x=e^{-1.5}$ or $x=0.223$	• Correct $f'(x)$.	• Correct $f'(x)$. AND Correct $f''(x)$. AND Correct x-value.	

2023 Question 3d.

(d)	$f(x)=x^{-1}-2x^{-3}$ $f'(x)=-x^{-2}+6x^{-4}=0$ $\frac{6}{x^4}=\frac{1}{x^2}$ $6x^2-x^4=0$ $x^2(6-x^2)=0$ $x\neq 0$ *so* $x=\pm\sqrt{6}$ $f''(x)=2x^{-3}-24x^{-5}$ $f''(\sqrt{6})=-0.136<0$ i.e. maximum $f''(-\sqrt{6})=0.136>0$ i.e. minimum Maximum at $\left(\sqrt{6},\frac{\sqrt{6}}{9}\right)=(2.45,0.2722)$ Minimum at $\left(-\sqrt{6},\frac{-\sqrt{6}}{9}\right)=(-2.45,-0.2722)$	• Correct values of x found, with evidence of derivative.	• Co-ordinates and nature of the two turning points found and distinguished, with evidence of a calculus method.	

2023 Question 3e.

(d)	$f(x) = x^{-1} - 2x^{-3}$ $f'(x) = -x^{-2} + 6x^{-4}$ $= \frac{-1}{x^2} + \frac{6}{x^4}$ $f'(x) = 0 \Rightarrow \frac{-1}{x^2} + \frac{6}{x^4} = 0$ $x^4 - 6x^2) = 0$ $x^2(x^2 - 6) = 0$ $x = 0$ not possible $x^2 - 6 = 0$ $x = \pm\sqrt{6}$ or $x = \pm 2.45$ **Second derivative test :** $f''(x) = 2x^{-3} - 24x^{-5}$ $= \frac{2}{x^3} - \frac{24}{x^5}$ $f''(\sqrt{6}) = -0.136 = -\frac{\sqrt{6}}{18}$ Since $f''(\sqrt{6}) < 0$, $x = \sqrt{6}$ is a local maximum. $f''(\sqrt{6}) = -0.136 = -\frac{\sqrt{6}}{18}$ Since $f''(\sqrt{6}) > 0$, $x = -\sqrt{6}$ is a local minimum. Maximum turning point when $x = \sqrt{6}$. Minimum turning point when $x = -\sqrt{6}$.	• Correct derivative. AND Correct two values of x found (not $x = 0$).	• x-coordinates of both stationary points found. AND The nature of the two turning points found with a correct first or second derivative test. Not required: $f\left(\sqrt{6}\right) = \frac{\sqrt{6}}{9}$ $= 0.272$ $f\left(-\sqrt{6}\right) = \frac{\sqrt{6}}{9}$ $= -0.272$	

2022 Question 1b.

(b)	$f(x) = \frac{x^2 + 1}{x}$ $f'(x) = \frac{x.(2x) - (x^2 + 1)}{x^2}$ $\frac{x.(2x) - (x^2 + 1)}{x^2} = 0$ $2x^2 - x^2 - 1 = 0$ $x^2 = 1$ $x = \pm 1$ OR $f(x) = x + x^{-1}$ $f'(x) = 1 - x^{-2}$ $1 - \frac{1}{x^2} = 0$ $x^2 - 1 = 0$ $x = \pm 1$	Correct solution with correct derivative Must have both solutions: $x = \pm 1$		

2022 Question 1e.

(e)	$y = e^{px^2}$ $\frac{dy}{dx} = 2pxe^{px^2}$ $\frac{d^2y}{dx^2} = 2px.2px.e^{px^2} + 2p.e^{px^2}$ $= 2pe^{px^2}(2px^2 + 1)$ At a point of inflection, $\frac{d^2y}{dx^2} = 0$ $2pe^{px^2}(2px^2 + 1) = 0$ **Equation 1** $2pe^{px^2} = 0$ $pe^{px^2} = 0$ $px^2 = \ln 0$ No solution as ln 0 is defined. OR $2pe^{px^2} = 0$ has no solutions because $2pe^{px^2} > 0$ for all values of x since $e^{px^2} > 0$ for all values of a and p is a positive **Equation 2** $2px^2 + 1 = 0$ $x^2 = \frac{-1}{2p}$ $x = \sqrt{\frac{-1}{2p}}$ $2px^2 + 1 = 0$ has no real solutions because p is a positive real constant, $\frac{-1}{2p}$ is negative and there is not a real solution when you take the square root of a negative number. OR $2px^2 + 1 = 0$ has no real solutions because $2px^2 + 1 = 0$ is always greater than zero because p is a positive real constant and x^2 is always greater than or equal to zero OR $2px^2 + 1 = 0$ has no real solutions because the discriminant is less than zero. $b^2 - 4ac = 0 - 4(2p)(1) = -8p$ Since p is a positive real constant. Therefore, there are no solutions to $\frac{d^2y}{dx^2} = 0$ nd	Correct $\frac{dy}{dx}$.	Correct $\frac{d^2y}{dx^2}$.	**T1** Correct $\frac{d^2y}{dx^2}$ with one part of the equation set to zero and the reason for there being no real solutions given for EITHER $2pe^{px^2} = 0$ OR $2px^2 + 1 = 0$ **T2** Correct proof with correct derivatives Both parts of the equation set to zero and the reason for there being no real solutions given for both equations

2022 Question 2c.

(c)	$d(t)=\frac{t^2-6}{2t^3}$ $v(t)=\frac{2t^3(2t)-(t^2-6)(6t^2)}{4t^6}$ $v(t)=\frac{36t^2-2t^4}{4t^6}$ $v(t)=\frac{18-t^2}{2t^4}$ Stationary point when $v(t)=0$ $18-t^2=0$ $t=\sqrt{18}\ (=4.24)$	Correct derivative.	Correct solution with correct derivative.	

2022 Question 2d.

(d)	$y=6e^{1-0.5x}$ $\text{Area}=6xe^{1-0.5x}$ $A'(x)=6e^{1-0.5x}+6xe^{1-0.5x}\times -0.5$ $=6e^{1-0.5x}-3xe^{1-0.5x}$ $=3e^{1-0.5x}(2-x)$ At maximum, $A'(x)=0$ $x=2$ $\text{Area}=12e^{1-1}=12$	Correct derivative of $A(x)$.	Correct solution with correct derivative.	

2022 Question 3d.

(d)	$y = 9x - 2 + \frac{3}{3x-1}$ $\frac{dy}{dx} = 9 - 3(3x-1)^{-2} \times 3$ $= 9 - \frac{9}{(3x-1)^2}$ Stationary point $\frac{dy}{dx} = 0$ $9 - \frac{9}{(3x-1)^2} = 0$ $9 = \frac{9}{(3x-1)^2}$ $(3x-1)^2 = 1$ $3x - 1 = \pm 1$ $x = \frac{1 \pm 1}{3}$ $x = 0$ or $x = \frac{2}{3}$ $\frac{d^2y}{dx^2} = \frac{54}{(3x-1)^3}$ $x = 0 \quad \frac{d^2y}{dx^2} = \frac{54}{(-1)^3} < 0$ Local max at $x = 0$ $x = \frac{2}{3} \quad \frac{d^2y}{dx^2} = \frac{54}{(1)^3} > 0$ Local min at $x = \frac{2}{3}$	Correct derivative.	Correct solution with correct derivative. The nature of each turning point stated but not determined using a calculus method.	**T1** Correct solution with correct derivative. The nature of each turning point determined with a first or second derivative test.

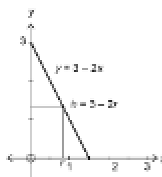
2022 Question 3e.

(e)	Total time = time (HP) + time (PS) **Method A** Let x = distance PQ $T=\frac{4-x}{10}+\frac{\sqrt{x^2+4}}{6}$ $\frac{dT}{dx}=\frac{-1}{10}+\frac{\frac{1}{2}(x^2+4)^{\frac{-1}{2}}.2x}{6}$ $\frac{dT}{dx}=\frac{-1}{10}+\frac{x}{6\sqrt{x^2+4}}$ For maximum/minimum time, $\frac{dT}{dx}=0$ $\frac{1}{10}=\frac{x}{6\sqrt{x^2+4}}$ $6\sqrt{x^2+4}=10x$ $\sqrt{x^2+4}=\frac{10}{6}x$ $x^2+4=\frac{25}{9}x^2$ $4=\frac{16}{9}x^2$ $\frac{36}{16}=x^2$ $x=1.5$ $4-1.5=2.5$ Megan should travel 2.5 km along the path before cutting across the park. **Method B** Let x = distance HP $T=\frac{x}{10}+\frac{\sqrt{\left((4-x)^2+4\right)}}{6}$ $\frac{dT}{dx}=\frac{1}{10}+\frac{(x-4)}{6\sqrt{x^2-8x+20}}$ $\frac{dT}{dx}=0$ $\frac{1}{10}+\frac{(x-4)}{6\sqrt{x^2-8x+20}}=0$ $5(x-4)=-3\sqrt{x^2-8x+20}$ $25\left(x^2-8x+16\right)=9\left(x^2-8x+20\right)$ $25x^2-200x+400=9x^2-72x+180$ $16x^2-128x+220=0$ $x=2.5$ or 5.5 Since $x<4$, $x=2.5$ km		Correct $\frac{dT}{dx}$.	**T1** Method A $x = 1.5$ found with correct derivative OR **T1** Method B $x = 2.5$ or 5.5 found (5.5 not discarded) with correct derivative. **T2** Correct solution with correct derivative.

2021 Question 1c.

(c)	$y=(2x+3)e^{x^2}$ $\frac{dy}{dx}=2e^{x^2}+(2x+3)(2x)e^{x^2}$ $\frac{dy}{dx}=2e^{x^2}(1+x(2x+3))$ $\frac{dy}{dx}=2e^{x^2}(2x^2+3x+1)$ $\frac{dy}{dx}=0$ for stationary points. $2e^{x^2}=0$ has no solutions since $2e^{x^2}>0$ $2x^2+3x+1=0$ $x=-\frac{1}{2}$ *or* $x=-1$	Correct derivative.	Correct solution with correct derivative.	

2021 Question 1e.

(e)	$V=\pi r^2 h$ $=\pi r^2(3-2r)$ $=3\pi r^2-2\pi r^3$ $\frac{dV}{dr}=6\pi r-6\pi r^2$ At maximum, $\frac{dV}{dr}=0$ $6\pi r(1-r)=0$ $r=0$ (no) $\therefore r=1$ $V=\pi 1^2(3-2\times1)=\pi$ $\frac{d^2V}{dr^2}=6\pi-12\pi r$ When $r=1$, $\frac{d^2V}{dr^2}=-6\pi<0$ Therefore $V=\pi$ is maximum volume.  	Correct expression for $\frac{dV}{dr}$.	Correct expression for $\frac{dV}{dr}$ and finds $r = 1$.	**T1**: Correct expression for $\frac{dV}{dr}$ and shows that $V=\pi$ but does not prove it is the maximum volume with either the first or second derivative test. **T2**: Correct expression for $\frac{dV}{dr}$ and correct proof.

2021 Question 2b.

(b)	$\frac{dy}{dx}=\frac{(x+1)2x-x^2}{(x+1)^2}$ $=\frac{x^2+2x}{(x+1)^2}$ $\frac{dy}{dx}=0\Rightarrow x(x+2)=0$ $x=0$ or $x=-2$	Correct solutions with correct derivative.		

2021 Question 3c.

(c)	$\frac{dy}{dx} = \frac{(x^2+4)-x(2x)}{(x^2+4)^2}$ $= \frac{4-x^2}{(x^2+4)^2}$ Increasing when $\frac{dy}{dx} > 0$ $\frac{4-x^2}{(x^2+4)^2} > 0$ $4-x^2 > 0$ $-2 < x < 2$	Correct $\frac{dy}{dx}$	Correct $\frac{dy}{dx}$ and identifies -2 and 2 as the boundaries of the interval required.	**T1:** Correct solution with correct derivative.

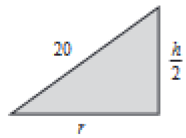
2021 Question 3e.

(e)	$\cos\theta = \frac{h}{S}$ $S^2 = h^2 + r^2$ $S = \sqrt{h^2 + r^2}$ k and r are constant $I = \frac{k\cos\theta}{S^2}$ $I = \frac{k\frac{h}{S}}{S^2}$ $= \frac{kh}{S^3}$ $I = \frac{kh}{\left(h^2 + r^2\right)^{\frac{3}{2}}}$ $\frac{dI}{dh} = \frac{\left(h^2 + r^2\right)^{\frac{3}{2}}k - kh\left(\frac{3}{2}\right)\left(h^2 + r^2\right)^{\frac{1}{2}}(2h)}{\left(h^2 + r^2\right)^3}$ $\frac{dI}{dh} = \frac{k\left(h^2 + r^2\right)^{\frac{3}{2}} - 3kh^2\left(h^2 + r^2\right)^{\frac{1}{2}}}{\left(h^2 + r^2\right)^3}$ $\frac{dI}{dh} = \frac{k\left(h^2 + r^2\right)^{\frac{1}{2}}\left(h^2 + r^2 - 3h^2\right)}{\left(h^2 + r^2\right)^3}$ $\frac{dI}{dh} = \frac{k\left(r^2 - 2h^2\right)}{\left(h^2 + r^2\right)^{\frac{5}{2}}}$ $\frac{dI}{dh} = 0 \Rightarrow k\left(r^2 - 2h^2\right) = 0$ $2h^2 = r^2$ $h^2 = \frac{r^2}{2}$ $h = \frac{r}{\sqrt{2}}$		Correct expression for $\frac{dI}{dh}$	T2: Correct proof with correct derivative

2020 Question 1c.

(c)	$\frac{dy}{dx} = \frac{(1 + \ln x)\cdot 1 - x\cdot\frac{1}{x}}{(1 + \ln x)^2}$ $= \frac{\ln x}{(1 + \ln x)^2}$ $\frac{dy}{dx} = 0 \Rightarrow \ln x = 0$ $x = 1$	Correct derivative.	Correct solution with correct derivative.	

2020 Question 1e.

(e)	$r^2 + \left(\frac{h}{2}\right)^2 = 400$ $r^2 = 400 - \frac{h^2}{4}$ $V_{\text{cyl}} = \pi r^2 h$ $= \pi\left(400 - \frac{h^2}{4}\right)h$ $= \pi\left(400h - \frac{h^3}{4}\right)$ $\frac{dV}{dh} = \pi\left(400 - \frac{3h^2}{4}\right)$ $\frac{dV}{dh} = 0 \Rightarrow 400 - \frac{3h^2}{4} = 0$ $h = \sqrt{\frac{1600}{3}} = \frac{40}{\sqrt{3}} = 23.1\text{ cm}$ $r = 16.3\text{ cm}$ $V = \pi \times 16.3^2 \times 23.1$ $= 19\,300\text{ cm}^3$ $V = 19\,347\text{ cm}^3$  	Correct expression for $\frac{dV}{dh}$ or $\frac{dV}{dr}$	Correct value of r or h with correct derivatives. Units not required.	Correct solution with correct derivatives. Units not required.

2020 Question 2c.

(c)	$f'(x) = (2x-3)2xe^{x^2+k} + 2e^{x^2+k}$ $= e^{x^2+k}\left((2x-3)2x+2\right)$ $= e^{x^2+k}\left(4x^2-6x+2\right)$ $= 2e^{x^2+k}\left(2x^2-3x+1\right)$ $f'(x) = 0 \Rightarrow 2e^{x^2+k} = 0$ or $2x^2-3x+1=0$ $2e^{x^2+k}$ has no solutions since $2e^{x^2+k}$ is always positive. $2x^2-3x+1=0$ $(2x-1)(x-1)=0$ $x = \frac{1}{2}$ or $x = 1$	Correct derivative.	Correct solution with correct derivative. Reference to $2e^{x^2+k} = 0$ is not required	

2020 Question 3d.

(d)	$y=(x-3)^{-1}+x$ $\frac{dy}{dx}=-1(x-3)^{-2}+1$ $=\frac{-1}{(x-3)^2}+1$ $\frac{dy}{dx}=0 \Rightarrow x-3=\pm 1$ $x=2$ or 4 $\frac{d^2x}{dy^2}=\frac{2}{(x-3)^3}$ $x=2 \Rightarrow \frac{d^2x}{dy^2}<0$ Local max at $x=2$ $x=4 \Rightarrow \frac{d^2x}{dy^2}>0$ Local min at $x=4$	Correct expression for $\frac{dy}{dx}$.	Correct expressions for $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$ OR Correct expression for $\frac{dy}{dx}$ plus x-coordinates of TPs found and nature stated without correct use of first or second derivative test.	Correct solution with correct derivatives. With use of the first derivative test or second derivative test to justify the nature of the turning points.

2019 Question 1d.

(d)	$\frac{dy}{dx}=3x^2e^x+x^3e^x$ $=x^2e^x(3+x)$ $\frac{dy}{dx}<0$ $\Rightarrow x^2e^x(3+x)<0$ $3+x<0$ $x<-3$	Correct derivative.	Correct solution with correct derivative.	

2019 Question 3c.

(c)	$A(x) = x\left(4-\sqrt{x}\right)$ $= 4x - x^{\frac{3}{2}}$ $A'(x) = 4 - \frac{3}{2}x^{\frac{1}{2}}$ Maximum area when $A'(x) = 0$ $\frac{3}{2}\sqrt{x} = 4$ $\sqrt{x} = \frac{8}{3}$ $x = \frac{64}{9}$ Area $= \frac{64}{9}\left(4-\frac{8}{3}\right)$ $= \frac{64}{9}\times\frac{4}{3}$ $= \frac{256}{27} \quad \left(= 9\frac{13}{27}\right)$ Accept 9.48	Correct expression for $A'(x)$.	Correct solution with correct derivative.	

2019 Question 3d.

(d)	$a(t) = 2e^{t} - 8e^{-t}$ $a(t) = 0$ $\Rightarrow 2e^{t} - 8e^{-t} = 0$ $2e^{t} = 8e^{-t}$ $e^{2t} = 4$ $2t = \ln 4$ $t = \frac{1}{2}\ln 4 \quad (= \ln 2 = 0.693)$	Correct derivative.	Correct solution with correct derivative.	

2018 Question 2d.

(d)	$\frac{dy}{dx} = e^{x}\left(2x^2 - x - 1\right) + e^{x}(4x-1)$ $= e^{x}\left(2x^2 + 3x - 2\right)$ $\frac{dy}{dx} = 0 \Rightarrow e^{x}\left(2x^2+3x-2\right) = 0$ $\Rightarrow 2x^2 + 3x - 2 = 0$ $\Rightarrow (x+2)(2x-1) = 0$ $x = -2$ or $x = \frac{1}{2}$ Note $e^{x} = 0$ has no solutions since $e^{x} > 0 \forall x \varepsilon R$	Correct derivative.	Correct solution with correct derivative. Reference to $e^{x} = 0$ not required.	

2018 Question 3c.

(c)	$\text{Area} = \frac{1}{2}\cdot 2x\cdot\left(15-x^2\right) = 15x - x^3$ $\frac{dA}{dx} = 15 - 3x^2$ Max when $\frac{dA}{dx} = 0$ $3\left(5-x^2\right) = 0$ $x = \pm\sqrt{5}$ $y = 10$ $\text{Area} = \frac{1}{2}\times 2\sqrt{5}\times 10$ $= 10\sqrt{5}\ (= 22.36)$	Correct $\frac{dA}{dx}$	Correct solution with correct derivative.	

2018 Question 3e.

(e)	$w^2 = 5^2 + \left(5 - \frac{x}{2}\right)^2$ $w^2 = 25 + 25 - 5x + 0.25x^2$ $w^2 = 0.25x^2 - 5x + 50$ $w = \left(0.25x^2 - 5x + 50\right)^{\frac{1}{2}}$ $\text{Length} = x + 4w$ $= x + 4\left(0.25x^2 - 5x + 50\right)^{\frac{1}{2}}$ $\frac{dL}{dx} = 1 + 2\left(0.25x^2 - 5x + 50\right)^{\frac{-1}{2}} \times \left(0.5x - 5\right)$ $\frac{dL}{dx} = 1 + \frac{x-10}{\left(0.25x^2 - 5x + 50\right)^{\frac{1}{2}}}$ For max/min $\frac{dL}{dx} = 0$ $\frac{x-10}{\left(0.25x^2 - 5x + 50\right)^{\frac{1}{2}}} = -1$ $x - 10 = -1\left(0.25x^2 - 5x + 50\right)^{\frac{1}{2}}$ $\left(x-10\right)^2 = 0.25x^2 - 5x + 50$ $x^2 - 20x + 100 = 0.25x^2 - 5x + 50$ $0.75x^2 - 15x + 50 = 0$ $x = 15.77$ not applicable $x = 4.23\text{ cm}$		Correct expression for $\frac{dL}{dx}$	Correct solution with correct derivative.

2017 Question 1e.

(e)	$\frac{dy}{dx}=\frac{(x^2-1).a-(ax-b).2x}{(x^2-1)^2}$ At $x=3$, $\frac{dy}{dx}=0 \Rightarrow 8a-(3a-b)\times 6=0$ $-10a+6b=0$ $5a=3b$	Correct derivative.	Correct derivative plus one of the two equations relating a and b.	Correct solution with correct derivative.
	The curve passes through (3,1) $\Rightarrow 1=\frac{3a-b}{8}$ $8=3a-b$ $24=9a-3b$ $=9a-5a$ $=4a$ $\therefore a=6$ and $b=10$			

2017 Question 2b.

(b)	$P(w)=96\ln(w+1.25)-16w-12$ $\frac{dP}{dw}=\frac{96}{w+1.25}-16$ Maximum when $\frac{dP}{dw}=0$ $\frac{96}{w+1.25}-16=0$ $96=16(w+1.25)$ $76=16w$ $w=4.75$	Correct solution with correct derivative.		

2017 Question 2d.

(d)	$d^2=(4-x)^2+(\sqrt{x})^2$ $=16-8x+x^2+x$ $=16-7x+x^2$ Minimum distance $\Rightarrow \frac{\mathrm{d}(d^2)}{\mathrm{d}x}=0$ $\frac{\mathrm{d}(d^2)}{\mathrm{d}x}=-7+2x=0$ $x=3.5$ $y=\sqrt{x}=\sqrt{3.5}$ $P=(3.5,\sqrt{3.5})$	Correct expression for $\frac{\mathrm{d}d}{\mathrm{d}x}$ or $\frac{\mathrm{d}(d^2)}{\mathrm{d}x}$	Correct solution with correct derivative.	

	Alternative: $d=(x^2-7x+16)^{\frac{1}{2}}$ $\frac{\mathrm{d}d}{\mathrm{d}x}=\frac{1}{2}(x^2-7x+16)^{\frac{-1}{2}}.(2x-7)$ $=\frac{2x-7}{2\sqrt{x^2-7x+16}}$ Minimum when $\frac{\mathrm{d}d}{\mathrm{d}x}=0$ $2x-7=0$ etc			

2017 Question 2e.

(e)	$\text{Area} = 2x\sqrt{r^2 - x^2}$ $A(x) = 2x(r^2 - x^2)^{\frac{1}{2}}$ $A'(x) = 2(r^2 - x^2)^{\frac{1}{2}} + 2x.\frac{1}{2}(r^2 - x^2)^{\frac{-1}{2}}.(-2x)$ $= 2\sqrt{r^2 - x^2} - \frac{2x^2}{\sqrt{r^2 - x^2}}$ $A'(x) = 0 \Rightarrow \sqrt{r^2 - x^2} = \frac{x^2}{\sqrt{r^2 - x^2}}$ $r^2 - x^2 = x^2$ $2x^2 = r^2$ $x^2 = \frac{r^2}{2}$ $x = \frac{r}{\sqrt{2}}$		Correct derivative.	Correct solution presented in a correct mathematical manner.

2016 Question 1e.

(e)	$f(x) = e^{-(x-k)^2}$ $f'(x) = -2(x-k)e^{-(x-k)^2}$ $f''(x) = -2e^{-(x-k)^2} + 4(x-k)^2 e^{-(x-k)^2}$ $= e^{-(x-k)^2}\left[4(x-k)^2 - 2\right]$ $f''(x) = 0 \Rightarrow 4(x-k)^2 - 2 = 0$ $4(x-k)^2 = 2$ $(x-k)^2 = \frac{1}{2}$ $(x-k) = \frac{\pm 1}{\sqrt{2}}$ $x = k \pm \frac{1}{\sqrt{2}}$	Correct $f'(x)$	Correct $f''(x)$	Correct solutions with correct $f'(x)$ and $f''(x)$

2016 Question 2e.

(e)	$Vol = \frac{1}{3}\pi r^2 h$ $h = 6 + s$ $s^2 + r^2 = 6^2$ $r^2 = 36 - s^2$ $\therefore V = \frac{1}{3}\pi(36 - s^2)(6 + s)$ $= \frac{1}{3}\pi(216 + 36s - 6s^2 - s^3)$ $\frac{dV}{ds} = \frac{1}{3}\pi(36 - 12s - 3s^2)$ Max volume when $\frac{dV}{ds} = 0$ $\Rightarrow 3s^2 + 12s - 36 = 0$ $s^2 + 4s - 12 = 0$ $(s+6)(s-2) = 0$ $s = -6$ or $s = 2$ $s = 2$		Correct expression for $\frac{dV}{ds}$	Correct solution.

2016 Question 3c.

(c)	$Area = A(x) = x(x-6)^2$ $= x^3 - 12x^2 + 36x$ $A'(x) = 3x^2 - 24x + 36$ Max when $A'(x) = 0$ $3(x^2 - 8x + 12) = 0$ $3(x-6)(x-2) = 0$ Max when $x = 2$ Max Area = $2 \times 16 = 32$	Correct expression for $A'(x)$	Correct solution for maximum area with correct derivative.	

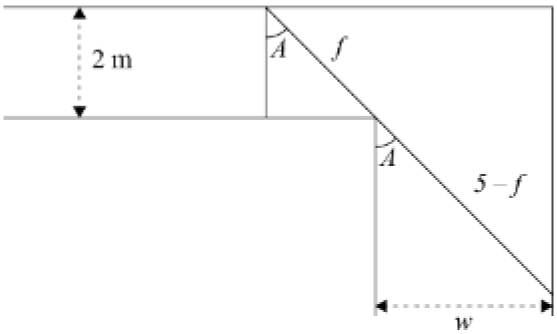
2015 Question 1c.

(c)	$f'(x) = 8 - \frac{2}{(x+1)^2}$ $f'(x) > 0 \Rightarrow 8 > \frac{2}{(x+1)^2}$ $(x+1)^2 > \frac{1}{4}$ Either $x + 1 > \frac{1}{2}$ or $x + 1 < \frac{-1}{2}$ $x > \frac{-1}{2}$ or $x < \frac{-3}{2}$	Correct derivative.	Correct solution with correct derivative.	

2015 Question 3b.

(b)	$f'(x)=\frac{e^{3x}-x.3e^{3x}}{\left(e^{3x}\right)^2}$ $=\frac{1-3x}{e^{3x}}$ $f'(x)=0\Rightarrow x=\frac{1}{3}$	Correct solution with correct derivative.		

2015 Question 3e.

(e)	 $\cos A=\frac{2}{f}$ $f=\frac{2}{\cos A}$ $\sin A=\frac{w}{5-f}$ $w=(5-f)\sin A$ $=\left(5-\frac{2}{\cos A}\right)\sin A$ $=5\sin A-2\tan A$ $\frac{dw}{dA}=5\cos A-2\sec^2 A$ $\frac{dw}{dA}=0\Rightarrow 5\cos A-2\sec^2 A=0$ $5\cos^3 A-2=0$ $\cos^3 A=\frac{2}{5}$ $A=42.5°$ $w=1.55$ m	Differentiate corretedly related but incorrect expression for w.	Correct $\frac{dw}{dA}$	Correct solution.

2014 Question 1e.

(e)	$h^2 + r^2 = 400$ $h = \sqrt{400 - r^2}$ $V = \frac{1}{3}\pi r^2 h = \frac{1}{3}\pi r^2 \sqrt{400 - r^2}$ $\frac{dV}{dr} = \frac{2}{3}\pi r\sqrt{400 - r^2} + \frac{1}{3}\pi r^2 . \frac{1}{2}(400 - r^2)^{\frac{-1}{2}} . -2r$ $\frac{dV}{dr} = \frac{\frac{2}{3}\pi r(400 - r^2) - \frac{1}{3}\pi r^3}{\sqrt{400 - r^2}}$ At maximum volume: $\frac{dV}{dR} = 0$ $2(400 - r^2) = r^2$ $3r^2 = 800$ $r = 16.3$ cm	Correct derivative for an incorrect but relevant expression for V.	A correct expression $\frac{dV}{dr}$ for .	A correct solution. Units not required.

	$V = 3225$ cm^3 Alternative working: $r^2 = 400 - h^2$ $V = \frac{1}{3}\pi r^2 h = \frac{1}{3}\pi(400 - h^2)h$ $= \frac{\pi}{3}(400h - h^3)$ $\frac{dV}{dh} = \frac{\pi}{3}(400 - 3h^2)$ At maximum, $\frac{dV}{dh} = 0$ $400 - 3h^2 = 0$ $h^2 = \frac{400}{3}$ $h = \frac{20}{\sqrt{3}} = 11.547$ cm $V = 3225$ cm^3			

2014 Question 2d.

(d)	$C = 4v + \frac{1000000}{v}$ $\frac{dC}{dv} = 4 - \frac{1000000}{v^2}$ Minimum when $\frac{dC}{dv} = 0$ $v^2 = 250000$ $v = 500$ $C = 4 \times 500 + \frac{1000000}{500} = 4000$	Correct value for *v* with correct derivative.	A correct solution. Units not required.	

2014 Question 3b.

(b)	$y = x + \frac{32}{x^2}$ $\frac{dy}{dx} = 1 - \frac{64}{x^3}$ Stationary points when $\frac{dy}{dx} = 0$ $\Rightarrow x^3 = 64$ $x = 4$	A correct solution.		

2014 Question 3c.

(c)	$f(x) = 5x - x\ln x$ $f'(x) = 5 - \ln x - \frac{x}{x}$ $= 4 - \ln x$ Increasing $\Rightarrow f'(x) > 0$ $4 - \ln x > 0$ $\ln x < 4$ $x < e^4$ $x < 54.6$ But if $x \leq 0$ then $\ln x$ is not defined, so $0 < x < 54.6$	A correct expression for the derivative.	A correct solution.	

2014 Question 3e.

(e)	$h = 40 - 2r$ $V = \pi r^2 h$ $= \pi r^2 (40 - 2r)$ $= 40\pi r^2 - 2\pi r^3$ $\frac{dV}{dr} = 80\pi r - 6\pi r^2$ $\frac{dV}{dr} = 0 \Rightarrow 80\pi r - 6\pi r^2 = 0$ $2\pi r(40 - 3r) = 0$ $r = \frac{40}{3}$ or 0 $r = \frac{40}{3}$ cm	Correct derivative for an incorrect but relevant expression for V.	A correct expression for $\frac{dV}{dr}$	A correct solution. Units not required.

2013 Question 1c.

(c)	$\frac{dy}{dx} = -2xe^{6-x^2}$ $\frac{d^2y}{dx^2} = -2e^{6-x^2} + 4x^2e^{6-x^2}$ Point of inflection when $\frac{d^2y}{dx^2} = 0$ $(4x^2 - 2)e^{6-x^2} = 0$ $4x^2 - 2 = 0$ $x = \pm\frac{1}{\sqrt{2}}$	Correct $\frac{dy}{dx}$	Correct solution with correct first and second derivatives. $\pm$ not required, accept positive answer only.	

2013 Question 1e.

(e)	$20 = 2\pi r^2 + 2\pi rh$ $2\pi r(r+h) = 20$ $h = \frac{10}{\pi r} - r$ $V = \pi r^2 h = \pi r^2 \cdot \left(\frac{10}{\pi r} - r\right)$ $V = 10r - \pi r^3$ $\frac{dV}{dr} = 10 - 3\pi r^2$ $\frac{dV}{dr} = 0 \Rightarrow r = \sqrt{\frac{10}{3\pi}}$ or $r = 1.03$ m OR $20 = \pi r^2 + 2\pi rh$ $V = 10r - \frac{\pi r^3}{2}$ $\frac{dV}{dr} = 10 - \frac{3\pi r^2}{2}$ $r = \sqrt{\frac{20}{3\pi}} = 1.46$		Equation for volume in terms of 1 variable found, and differentiated correctly.	Problem solved including correct derivative.

2013 Question 2c.

(c)	$f'(x) = 1 - e^x + \frac{k}{x^2}$ $f'(x) = 0 \Rightarrow 1 - e^{-1} + k = 0$ $k = e^{-1} - 1$ Or $k = -0.632$	Correct derivative.	Correct value for k and correct derivative.	

2013 Question 2e.

(e)	$A(\theta) = 64\sin\theta + 64\sin\theta\cos\theta$ OR $A(\theta) = 64\sin\theta + 32\sin 2\theta$ $A'(\theta) = 64\cos\theta + 64\cos^2\theta - 64\sin^2\theta$ OR $A'(\theta) = 64\cos\theta + 64\cos 2\theta$ $= 64\cos\theta + 64\cos^2\theta - 64(1-\cos^2\theta)$ $= 64(2\cos^2\theta + \cos\theta - 1)$ Minimum when $A'(\theta) = 0$ $2\cos^2\theta + \cos\theta - 1 = 0$ $(2\cos\theta - 1)(\cos\theta + 1) = 0$ Or $\cos\theta = \frac{1}{2}$ or $\cos\theta = -1$ (NO) $\theta = 60°$ or $\theta = \frac{\pi}{3}$		Correct derivative.	Correct solution with correct derivatives.

2013 Question 3b.

(b)	$f'(x)=1-16(x-2)^{-2}$ Turning point when $f'(x)=0$ $1=\frac{16}{(x-2)^2}$ $(x-2)^2=16$ $x=-2$ or $x=6$	Correct solution with correct derivative.		

2013 Question 3c.

(c)	$f'(x)=50-\left(30\ln 2x+30x\cdot\frac{1}{x}\right)$ $=20-30\ln 2x$ Maximum when $f'(x)=0$ $20=30\ln 2x$ $\frac{2}{3}=\ln 2x$ $x=\frac{e^{\frac{2}{3}}}{2}=0.974$	Correct derivative.	Correct solution with correct derivative.	