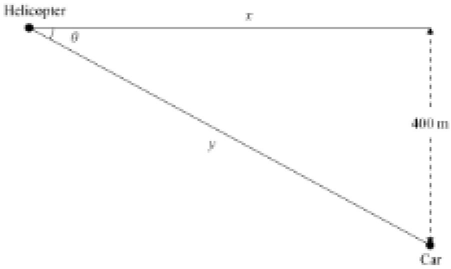
Differentiation Collated Past Answers – Rates of Change

2023 Question 1b.

(b)	$f'(t) = t^2\left(2e^{2t}\right) + e^{2t}(2t)$ $= 2t^2e^{2t} + 2te^{2t}$ $= 2te^{2t}(t+1)$ $f'(1.5) = 3e^3(2.5)$ $= 7.5e^3 = 150.64$	• Correct derivative. AND Correct rate of change.		

2023 Question 2e.

(e)	 Let x = horizontal distance between the helicopter and the car. Let y = direct distance between the helicopter and the car. Given: $\frac{d\theta}{dt} = 0.002\ rad\ s-1$ $\tan\theta = \frac{400}{x}$ $x = 400\cot\theta$ $\frac{dx}{d\theta} = -400\operatorname{cosec}^2\theta$ $= \frac{-400}{\sin^2\theta}$ $\frac{dx}{dt} = \frac{dx}{d\theta} \times \frac{d\theta}{dt}$ $= \frac{-400}{\sin^2\theta} \times 0.002$ $= \frac{-0.8}{\sin^2\theta}$ When $y = 2500$, $\sin\theta = \frac{400}{2500}$ $\theta = 0.1607$ rad $\frac{dx}{dt} = \frac{-0.8}{\sin^2(0.1607}$ $= 31.25$ When the helicopter is travelling at 72 m s^{-1}, The speed of the car $= 72 - 31.25$ $= 40.75$ m s^{-1} ($= 146.7$ km/hr)	• Finds $\frac{dx}{d\theta}$.	• Finds an expression for $\frac{dx}{dt}$.	T1 Finds the value for $\frac{dx}{dt} = -31.25$ With correct derivatives. OR Finds correct solution but with one minor error. T2 Finds $\frac{dx}{dt} = -31.25$ with correct derivatives. AND The speed of the car = 40.76 m s^{-1}.

2022 Question 3c.

(c)	$V=\pi\left(\frac{3}{2}h^2+3h\right)$ $\frac{dV}{dh}=\pi(3h+3)$ $\frac{dV}{dt}=20$ $\frac{dh}{dt}=\frac{dh}{dV}\times\frac{dV}{dt}$ $=\frac{1}{\pi(3h+3)}\times 20$ At $h=3, \frac{dh}{dt}=\frac{20}{12\pi}$ $=\frac{5}{3\pi}=0.531\text{ cm s}^{-1}$	Correct expressions for $\frac{dV}{dh}$ and $\frac{dV}{dt}$. $\frac{dV}{dt}$ can be implied by the expression for $\frac{dh}{dt}$.	Correct solution with correct derivative for $\frac{dh}{dt}$.	

2021 Question 2d.

(d)	$\frac{dV}{dt}=60$ $V=\frac{4}{3}\pi r^3$ $\frac{dV}{dr}=4\pi r^2$ $\frac{dr}{dt}=\frac{dV}{dt}\times\frac{dr}{dV}$ $=\frac{60}{4\pi r^2}$ $=\frac{15}{\pi r^2}$ $r=15\Rightarrow\frac{dr}{dt}=\frac{15}{\pi 15^2}$ $=\frac{1}{15\pi}(=0.0212)\text{ cm s}^{-1}$	Correct expression for $\frac{dr}{dt}$.	Correct solution with correct $\frac{dr}{dt}$.	

2020 Question 2b.

(b)	$\frac{dV}{dt}=-4250e^{-0.25t}-1000e^{-0.5t}$ $t=8\Rightarrow\frac{dV}{dt}=-4250e^{-2}-1000e^{-4}$ $=-593.50$ Decreasing at \$593.50 per year.	Correct solution with correct derivative. Units not required. Interpretation not required.		

2020 Question 2d.

(d)	$\tan\theta = \frac{h}{500}$ $h = 500\tan\theta$ $\frac{dh}{d\theta} = 500\sec^2\theta = \frac{500}{\cos^2\theta}$ $t = 10$ $\tan\theta = \frac{480}{500}$ $\theta = 0.765$ $\frac{d\theta}{dt} = \frac{dh}{dt} \times \frac{d\theta}{dh}$ $= 9.6t \times \frac{\cos^2\theta}{500}$ $= 96 \times \frac{\cos^2(0.765)}{500}$ $= 0.0999$ (accept 0.1)	Correct expression for $\frac{dh}{d\theta}$.	Correct expression for $\frac{d\theta}{dt}$.	Correct solution with correct derivatives.

2019 Question 1e.

(e)	$\frac{dV}{dt} = \frac{dS}{dt} \times \frac{dr}{dS} \times \frac{dV}{dr}$ $S = 4\pi r^2 \Rightarrow \frac{dS}{dr} = 8\pi r$ $V = \frac{4}{3}\pi r^3 \Rightarrow \frac{dV}{dr} = 4\pi r^2$ $\frac{dS}{dt} = 0.4$ when $r = 0.5$ $\frac{dV}{dt} = 0.4 \times \frac{1}{8\pi r} \times 4\pi r^2$ $= 0.2r$ When $r = 0.5$, $\frac{dV}{dt} = 0.1\,\text{m}^3/s$	Correct expressions for $\frac{dS}{dr}$ and $\frac{dV}{dr}$.	Correct expression for $\frac{dV}{dt}$. Anything equivalent. Line 5 is ok.	Correct solution with correct derivatives. Units not required.

2019 Question 2d.

(d)	$\frac{d\theta}{dt} = 0.01 \text{ rad / s}$ $\frac{dh}{dt} = \frac{d\theta}{dt} \times \frac{dh}{d\theta}$ $\sin\theta = \frac{h}{22}$ $h = 22\sin\theta$ $\frac{dh}{d\theta} = 22\cos\theta$ $\therefore \frac{dh}{dt} = 0.22\cos\theta$ $h = 15 \Rightarrow \theta = \sin^{-1}\left(\frac{15}{22}\right) = 0.75$ $\frac{dh}{dt} = 0.22\cos(0.75) = 0.16 \text{ m s}^{-1}$	Correct expression for $\frac{dh}{d\theta}$.	Correct solution with correct derivative, $\frac{dh}{d\theta}$. Units not required.	

2018 Question 1d.

(d)	$\frac{dL}{dt} = 0.6 \text{ m s}^{-1}$ $L^2 = x^2 + 3^2$ $x = \sqrt{L^2 - 9}$ $\frac{dx}{dL} = \frac{1}{2}(L^2 - 9)^{\frac{-1}{2}} \cdot 2L$ $= \frac{L}{\sqrt{L^2 - 9}}$ $\frac{dx}{dt} = \frac{dL}{dt} \times \frac{dx}{dL}$ $= 0.6 \times \frac{L}{\sqrt{L^2 - 9}}$ When $L = 5.4$ $\frac{dx}{dt} = 0.6 \times \frac{5.4}{\sqrt{5.4^2 - 9}}$ $= 0.722 \text{ m s}^{-1}$	Correct expression for $\frac{dx}{dL}$ or $\frac{dL}{dx}$.	Correct solution with correct derivatives.	

2018 Question 2e.

(e)	$\frac{dV}{dt}=150\text{ cm}^3/\text{s}$ $\frac{dSA}{dt}=\frac{dV}{dt}\times\frac{dr}{dV}\times\frac{dSA}{dr}$ $h=2.5r$ $V=\frac{1}{3}\pi r^2 h$ $=\frac{5}{6}\pi r^3$ $\frac{dV}{dr}=2.5\pi r^2$ $SA=\pi r^2$ $\frac{dSA}{dr}=2\pi r$ $\frac{dSA}{dt}=150\times\frac{1}{2.5\pi r^2}\times 2\pi r$ $=\frac{120}{r}$ When $h=125\text{ cm}$, $r=50\text{ cm}$ $\frac{dSA}{dt}=\frac{120}{50}=2.4\text{ cm}^2/\text{s}$	Correct expression for $\frac{dV}{dr}$ in terms of one variable.	Correct expression for $\frac{dV}{dr}$ and $\frac{dSA}{dr}$ in terms of r, and an attempt to relate two (or more) derivatives.	Correct solution.

2017 Question 3d.

(d)	Let h = height above Sarah's eye level. $\tan\theta=\frac{h}{30}$ $h=30\tan\theta$ $\frac{dh}{d\theta}=30\sec^2\theta$ $\frac{dh}{dt}=2$ $\frac{d\theta}{dt}=\frac{dh}{dt}\times\frac{d\theta}{dh}$ $=2\times\frac{1}{30\sec^2\theta}$ $=\frac{\cos^2\theta}{15}$ At $h=20$ $\theta=\tan^{-1}\left(\frac{20}{30}\right)=0.588$ $\frac{d\theta}{dt}=\frac{(\cos 0.588)^2}{15}$ $=0.046$ radians per second	Correct expression for $\frac{dh}{d\theta}$	Correct solution with correct derivatives. Ignore units in the solution.	

2016 Question 1b.

(b)	$\frac{dh}{dt}=\frac{3.2\pi}{25}\cos\left(\frac{4\pi}{25}t+\frac{\pi}{2}\right)$ $=0.402\cos\left(\frac{36\pi}{25}+\frac{\pi}{2}\right)$ $=0.395$ metres per hour	Correct solution with correct derivative		

2016 Question 2d.

(d)	$\frac{dV}{dt}=4800\text{ cm}^3\text{ s}^{-1}$ $V=\frac{4}{3}\pi r^3$ $\frac{dV}{dr}=4\pi r^2$ $\frac{dr}{dt}=\frac{dr}{dV}\times\frac{dV}{dt}$ $=\frac{4800}{4\pi r^2}=\frac{1200}{\pi r^2}$ $V=288000\pi=\frac{4}{3}\pi r^3$ $288000=\frac{4}{3}r^3$ $r^3=216000$ $r=60$ cm $\therefore\frac{dr}{dt}=\frac{1200}{\pi\times 60^2}=0.106\text{ cm s}^{-1}$	Correct expression for $\frac{dr}{dt}$	Correct solution with correct $\frac{dr}{dt}$ – units not required.	

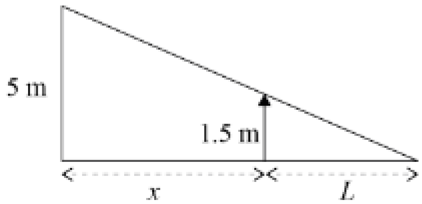
2016 Question 3e.

(e)	$\tan\alpha = \frac{15}{d}$ $\quad$ $\tan(\alpha+\theta) = \frac{20.4}{d}$ $\tan\theta = \tan((\alpha+\theta)-\alpha)$ $= \frac{\tan(\alpha+\theta) - \tan\alpha}{1 - \tan(\alpha+\theta)\cdot\tan\alpha}$ $= \frac{\frac{20.4}{d} - \frac{15}{d}}{1 + \frac{20.4\times 15}{d^2}}$ $= \frac{\frac{5.4}{d}}{\frac{d^2+306}{d^2}}$ $= \frac{5.4d}{d^2+306}$ Max when $\frac{\mathrm{d}(\tan\theta)}{\mathrm{d}d} = 0$ $\frac{(d^2+306)\times 5.4 - 5.4d\times 2d}{(d^2+306)^2} = 0$ $5.4d^2 + 306\times 5.4 - 10.8d^2 = 0$ $5.4d^2 - 306\times 5.4 = 0$ $d^2 = 306$ $d = 17.5$ m		Correct expression for $\frac{\mathrm{d}(\tan\theta)}{\mathrm{d}d}$ or $\frac{\mathrm{d}\theta}{\mathrm{d}d}$	Correct solution – units not required.

2015 Question 1e.

(e)	Let V = volume (m^3) S = slant height (m) h = height (m) r = radius (m) $\cos 30 = \frac{r}{S}$ $S = \frac{r}{\cos 30}$ $\frac{dS}{dr} = \frac{1}{\cos 30}$ $\tan 30 = \frac{h}{r}$ $h = r\tan 30$ $V = \frac{1}{3}\pi r^2 h$ $= \frac{1}{3}\pi r^3 \tan 30$ $\frac{dV}{dr} = \pi r^2 \tan 30$ $\frac{dS}{dt} = \frac{dS}{dr} \times \frac{dr}{dV} \times \frac{dV}{dt}$ $= \frac{1}{\cos 30} \times \frac{1}{\pi r^2 \tan 30} \times 2$ When $r = 10$ m, $\frac{dS}{dt} = \frac{1}{\cos 30} \times \frac{1}{\pi 10^2 \times \tan 30} \times 2$ $= 0.01273$ m / minute	$\frac{dS}{dr}$ or $\frac{dV}{dr}$ correct.	Valid statement of the relationship between rates.	Correct solution with correct derivatives.

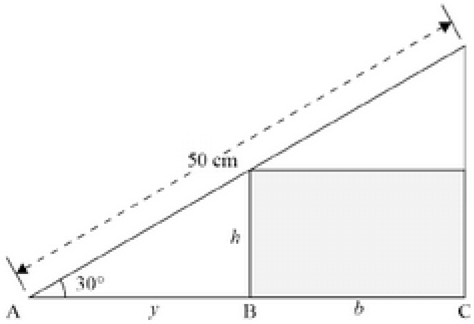
2015 Question 2d.

(d)	 $\frac{x+L}{5}=\frac{L}{1.5}$ $1.5x+1.5L=5L$ $1.5x=3.5L$ $x=\frac{7L}{3}$ $\frac{dx}{dL}=\frac{7}{3}$ $\frac{dx}{dt}=2$ $\frac{dL}{dt}=\frac{dL}{dx}\times\frac{dx}{dt}$ $=\frac{3}{7}\times 2$ $=\frac{6}{7}=0.857\text{ m s}^{-1}$	$\frac{dx}{dL}$ correct.		Correct solution with correct derivatives. (Units not required.)

2015 Question 2e.

(e)	Depth of water $= x$ $h=x+20$ $V=\frac{1}{3}h^3-\frac{1}{3}20^3$ $=\frac{1}{3}(x+20)^3-\frac{1}{3}20^3$ $\frac{dV}{dx}=(x+20)^2$ $A=(x+20)^2$ $\frac{dA}{dx}=2(x+20)$ $\frac{dV}{dt}=3000$ $\frac{dA}{dt}=\frac{dA}{dx}\times\frac{dx}{dV}\times\frac{dV}{dt}$ $=2(x+20)\times\frac{1}{(x+20)^2}\times 3000$ When $x=15$ $\frac{dA}{dt}=2\times 35\times\frac{1}{35^2}\times 3000=171.4\text{ cm}^2\text{ min}^{-1}$	Correct $\frac{dV}{dx}$ OR $\frac{dA}{dx}$	Correct $\frac{dV}{dx}$ AND $\frac{dA}{dx}$	Correct solution.

2014 Question 2e.

(e)	 $\tan 30 = \frac{h}{y}$ $h = y\tan 30$ $\cos 30 = \frac{y+b}{50}$ $y+b = 50\cos 30$ $b = 50\cos 30 - y$ Area = base × height $A = (50\cos 30 - y)(y\tan 30)$ $= 50y\sin 30 - y^2\tan 30$ $= 25y - \frac{y^2}{\sqrt{3}}$ $\frac{dA}{dy} = 25 - \frac{2y}{\sqrt{3}}$ At $y = 20$ $\frac{dA}{dy} = 25 - \frac{40}{\sqrt{3}}$ $\frac{dA}{dt} = \frac{dA}{dy} \times \frac{dy}{dt}$ $= \left(25 - \frac{40}{\sqrt{3}}\right) \times 3$ $= 5.72\ \text{cm}^2\,\text{s}^{-1}$	Correct derivative for an incorrect but relevant expression for A.	A correct expression for $\frac{dA}{dy}$	A correct solution. Units not Required.

2014 Question 3d.

(d)	$\frac{dh}{dt} = 1.5\ \text{m s}^{-1}$ $\tan\theta = \frac{h}{20}$ $h = 20\tan\theta$ $\frac{dh}{d\theta} = 20\sec^2\theta$ $\frac{d\theta}{dt} = \frac{d\theta}{dh} \times \frac{dh}{dt}$ $= \frac{1.5}{20\sec^2\theta}$ When $h = 20$, $\theta = \frac{\pi}{4}$, $\sec^2\theta = 2$ $\frac{d\theta}{dt} = \frac{1.5}{40} = 0.0375$ radians s^{-1}	A correct expression for $\frac{dh}{d\theta}$	A correct solution. Units not required.	

2013 Question 3e.

(e)	$\frac{dV}{dt} = 300$ $A = 4\pi r^2 \quad \Rightarrow \frac{dA}{dr} = 8\pi r$ $V = \frac{4}{3}\pi r^3 \quad \Rightarrow \frac{dV}{dr} = 4\pi r^2$ $\frac{dA}{dt} = \frac{dV}{dt} \cdot \frac{dA}{dr} \cdot \frac{dr}{dV}$ $= \frac{2400\pi r}{4\pi r^2}$ $= \frac{600}{r}$ $A = 7500 \quad \Rightarrow 4\pi r^2 = 7500$ $r = \sqrt{\frac{7500}{4\pi}} = 24.43 \text{ cm}$ $\therefore \frac{dA}{dt} = \frac{600}{24.43} = 24.56 \text{ cm}^2 \text{ s}^{-1}$	Correct expressions for $\frac{dV}{dr}$ and $\frac{dA}{dr}$	Correct expressions for $\frac{dV}{dr}$, $\frac{dA}{dr}$ and $\frac{dA}{dt}$	Correct solution along with correct expressions for $\frac{dV}{dr}$, $\frac{dA}{dr}$ and $\frac{dA}{dt}$