Differentiation Collated Past Papers - Tangents

2023 Question 1c.

(c)	$\frac{dy}{dx} = -6(x+1)^{-4}$ $= \frac{-6}{(x+1)^4}$ When $x = 1, \frac{dy}{dx} = -\frac{3}{8}$ or -0.375 $\frac{-6}{(x+1)^4} = -\frac{3}{8}$ $3(x+1)^4 = 48$ $(x+1)^4 = 16$ $x + 1 = \pm 2$ $x = 1$ or -3 $\therefore$ Second tangent touches the curve when $x = -3$	• Correct derivative for $\frac{dy}{dx}$. AND Correct gradient of $\frac{3}{8}$ found.	• Finds the correct value of x for the second tangent, with evidence of derivative.	

2023 Question 1d.

(d)	$x = 4\cos\theta$ and $y = 4\sin\theta$ $\frac{dx}{d\theta} = -4\sin\theta$ and $\frac{dy}{d\theta} = 4\cos\theta$ $\frac{dy}{dx} = \frac{4\cos\theta}{-4\sin\theta} = -\frac{x}{y} = -\frac{p}{q}$ OR Equation of circle: $x^2 + y^2 = 16$ Differentiating implicitly gives $2x + 2y\frac{dy}{dx} = 0$ $\frac{dy}{dx} = -\frac{x}{y}$ At (p,q), the gradient is $-\frac{p}{q}$ Equation of tangent: $y - q = -\frac{p}{q}(x - p)$ $qy - q^2 = -px + p^2$ $px + qy = p^2 + q^2$ as required.	• Correct derivative for $\frac{dy}{dx}$. • $\frac{dy}{dx}$ can be expressed in terms of θ or x,y or p,q).	• Proof completed, with correct .	

2023 Question 2c.

(c)	$f'(x)=\dfrac{(x^2+2x)e^x-e^x(2x+2)}{(x^2+2x)^2}$ $=\dfrac{e^x\left((x^2+2x)-(2x+2)\right)}{(x^2+2x)^2}$ $=\dfrac{e^x\left(x^2-2\right)}{(x^2+2x)^2}$ $f'(x)=0\Rightarrow e^x\left(x^2-2\right)=0$ $e^x\neq 0$ $x^2-2=0$ $x=\pm\sqrt{2}$ or $x=\pm 1.41$	• Correct derivative.	• Correct both values of x found, with evidence of derivative.	

2023 Question 3c.

(c)	$\dfrac{dx}{dt}=\sqrt{2}\pi\cos\left(\dfrac{\pi t}{5}\right)$ $\dfrac{dy}{dt}=\sqrt{2}\pi\sin\left(\dfrac{\pi t}{5}\right)\Rightarrow$ $\dfrac{dy}{dx}=\tan\left(\dfrac{\pi t}{5}\right)$ When $t=6.25\Rightarrow$ $\dfrac{dy}{dx}=\tan(1.25\pi)=1$ Normal gradient: $m=-1$	• Correct expression for $\dfrac{dx}{dt}$ AND $\dfrac{dy}{dt}$.	• Gradient of the normal found.	

2022 Question 1c.

(c)	$y=\sqrt{x+2}$ $\dfrac{dy}{dx}=\dfrac{1}{2}(x+2)^{\frac{-1}{2}}$ $=\dfrac{1}{2\sqrt{x+2}}$ At $x=0$, $\dfrac{dy}{dx}=\dfrac{1}{2\sqrt{2}}$ and $y=\sqrt{2}$ Equation of normal is $y=-2\sqrt{2}x+\sqrt{2}$ x intercept $(y=0)$ $\quad 0=-2\sqrt{2}x+\sqrt{2}$ $x=\dfrac{1}{2}$ Coordinate of P is $\left(\dfrac{1}{2},0\right)$	Correct derivative with $\dfrac{dy}{dx}$ evaluated at $x=0$.	Correct solution with correct $\dfrac{dy}{dx}$. Accept $x=\dfrac{1}{2}$. $y=0$ can be implied in the working.	

2022 Question 2b.

(b)	$y=\left(3x^2-2\right)^3$ $\frac{dy}{dx}=3\left(3x^2-2\right)^2(6x)$ At $x=2,\ \frac{dy}{dx}=3600$	Correct solution with correct derivative.		

2022 Question 2e.

(e)	$(y-5)^2=16(x-2)$ Method A $y-5=4\sqrt{x-2}$ $y=4\sqrt{x-2}+5$ $\frac{dy}{dx}=\frac{2}{\sqrt{x-2}}$ $\frac{dy}{dx}=1$ $\frac{2}{\sqrt{x-2}}=1$ $\sqrt{x-2}=2$ $x-2=4$ $x=6$ $y=13$ Method B $2(y-5)\frac{dy}{dx}=16$ $\frac{dy}{dx}=\frac{8}{y-5}$ $\frac{dy}{dx}=1$ $\frac{8}{y-5}=1$ $y=13$ $64=16x-32$ $x=6$ Equation of tangent $y-13=1(x-6)$ $y=x+7$ Axis intercepts: (0,7) and (-7, 0) Distance RS $=\sqrt{7^2+7^2}$ $=\sqrt{49\times 2}$ $=7\sqrt{2}$	Correct $\frac{dy}{dx}$.	Correct x and y values found (6,13) with correct $\frac{dy}{dx}$.	**T1** Finds equation of tangent and both axis intercepts with correct $\frac{dy}{dx}$. **T2** Correct solution with correct $\frac{dy}{dx}$.

2021 Question 2c.

(c)	$y=(x^2+3x+2)\cos 3x$ $\frac{dy}{dx}=(2x+3)\cos 3x-(x^2+3x+2)3\sin 3x$ Crosses y-axis $\Rightarrow x=0,\ y=2,\ \frac{dy}{dx}=3$ Normal gradient is $\frac{-1}{3}$ Equation of normal: $y-2=\frac{-1}{3}(x-0)$ $y=\frac{-1}{3}x+2$ $3y+x-6=0$	Correct derivative.	Correct solution with correct derivative.	

2021 Question 2e.

(e)	$y=\sqrt{2x-4}$ $\frac{dy}{dx}=\frac{1}{\sqrt{2x-4}}$ Gradient of tangent $=\frac{y-1}{x+2}$ $\frac{1}{\sqrt{2x-4}}=\frac{\sqrt{2x-4}-1}{x+2}$ $x+2=2x-4-\sqrt{2x-4}$ $\sqrt{2x-4}=x-6$ $2x-4=x^2-12x+36$ $x^2-14x+40=0$ $(x-4)(x-10)=0$ $x=4$ or $x=10$ Rejecting $x=4$ by checking the surd equation $x=10 \quad \sqrt{16}=4$ True $x=4 \quad \sqrt{4}=-2$ False One solution: $x=10$ Therefore, the coordinates of point P are $(10,4)$ OR Rejecting $x=4$ by checking the gradient: At $(10,4)$, $\frac{dy}{dx}=\frac{1}{\sqrt{16}}=\frac{1}{4}$ Gradient: $\frac{y-1}{x+2}=\frac{3}{12}=\frac{1}{4}$ At $(4,2)$, $\frac{dy}{dx}=\frac{1}{\sqrt{4}}=\frac{1}{2}$ Gradient: $\frac{y-1}{x+2}=\frac{1}{6}$ One solution: $x=10$ Therefore, the coordinates of point P are $(10,4)$	Correct derivative: $\frac{dy}{dx}=\frac{1}{\sqrt{2x-4}}$	Correct derivative: $\frac{dy}{dx}=\frac{1}{\sqrt{2x-4}}$ and $\sqrt{2x-4}=x-6$	**T1:** Correct solution with correct derivative: P (10,4) without any justification for $x \neq 4$ **T2:** Correct solution with correct derivative: P (10,4) $x \neq 4$ must be justified with respect to either the surd equation or the gradient of the tangent.

2021 Question 3b.

(b)	$\frac{dy}{dx}=\frac{2}{\sqrt{x}}-1$ At stationary point, derivative = 0. $\frac{2}{\sqrt{x}}=1$ $x=4$ Coordinates are $(4,6)$.	Correct solution with correct derivative.		

2021 Question 3d.

(d)	$\frac{dy}{dx} = \frac{(4x-k)4-(4x+k)4}{(4x-k)^2}$ $= \frac{16x-4k-16x-4k}{(4x-k)^2}$ $= \frac{-8k}{(4x-k)^2}$ When $x=3, \frac{dy}{dx} = \frac{-8}{27}$ $\frac{-8k}{(12-k)^2} = \frac{-8}{27}$ $\frac{k}{(12-k)^2} = \frac{1}{27}$ $27k = 144-24k+k^2$ $k^2-51k+144=0$ $k=48$ or $k=3$	Correct derivative.	Correct solution with correct derivative.	

2020 Question 1b.

(b)	$y = 3\sin 2x + \cos 2x$ $\frac{dy}{dx} = 6\cos 2x - 2\sin 2x$ At $x = \frac{\pi}{4}$ $\frac{dy}{dx} = 6\cos\frac{\pi}{2} - 2\sin\frac{\pi}{2} = -2$	Correct gradient with correct derivative.		

2020 Question 1d.

(d)	$\frac{dy}{dx} = x^2 \cdot -\sin x + 2x\cos x$ At $x=\pi$ $\frac{dy}{dx} = \pi^2 \cdot (-\sin\pi) + 2\pi\cos\pi$ $= -2\pi$ At $x=\pi$ $y = -\pi^2$ Tangent equation $y - y_1 = m(x-x_1)$ $y+\pi^2 = -2\pi(x-\pi)$ $y+\pi^2 = -2\pi x + 2\pi^2$ $y + 2\pi x = \pi^2$	Correct derivative.	Correct proof with correct derivative.	

2020 Question 3b.

(b)	$f(x)=2x-2\sqrt{x}$ $f'(x)=2-x^{\frac{-1}{2}}$ $\Rightarrow 2-\frac{1}{\sqrt{x}}=1$ $\frac{1}{\sqrt{x}}=1$ $x=1$	Correct value of x with correct derivative		

2020 Question 3c.

(c)	$y=(2x+1)^{\frac{1}{2}}$ $\frac{dy}{dx}=\frac{1}{2}(2x+1)^{\frac{-1}{2}}\times 2$ $=(2x+1)^{\frac{-1}{2}}$ $=\frac{1}{\sqrt{2x+1}}$ At $x=4$ $\frac{dy}{dx}=\frac{1}{3}$ Normal gradient $=-3$ $y-3=-3(x-4)$ $y=-3x+15$ x-intercept $\Rightarrow y=0$ $x=5$	Correct derivative.	Correct solution with correct derivative. Must have correct gradient of normal to justify $x=5$	

2019 Question 1c.

(c)	Quotient rule $\frac{dy}{dx}=\frac{(1+x^2)2e^{2x}-e^{2x}(2x)}{(1+x^2)^2}$ OR Product rule $\frac{dy}{dx}=e^{2x}(-2x)(1+x^2)^{-2}+(1+x^2)^{-1}(2e^{2x})$ When $x=2$, $\frac{dy}{dx}=\frac{6e^4}{25}$ or 13.1	Correct derivative.	Correct solution with correct derivative.	

2019 Question 3e.

(e)	$y = 2\sqrt{36-x^2}$ $\frac{dy}{dx} = \left(36-x^2\right)^{\frac{-1}{2}} . -2x$ $= \frac{-2x}{\sqrt{36-x^2}}$ Gradient of tangent $= \frac{-2\sqrt{36-x^2}}{(8-x)}$ $= \frac{2\sqrt{36-x^2}}{x-8}$ $\therefore \frac{2\sqrt{36-x^2}}{x-8} = \frac{-2x}{\sqrt{36-x^2}}$ $2\left(36-x^2\right) = 16x - 2x^2$ $72 - 2x^2 = 16x - 2x^2$ $72 = 16x$ $x = 4.5$ Or alternatively: $y = \frac{-2x}{\sqrt{36-x^2}}(x-8)$ $y = \frac{-2x}{\sqrt{36-x^2}} + \frac{16x}{\sqrt{36-x^2}}$ Substituting for y: $2\sqrt{36-x^2} = \frac{-2x^2}{\sqrt{36-x^2}} + \frac{16x}{\sqrt{36-x^2}}$ $2\left(36-x^2\right) = -2x^2 + 16x$ $36 - x^2 = -x^2 + 8x$ $36 = 8x$ $x = 4.5$	Correct $\frac{dy}{dx}$ of curve.	Correct $\frac{dy}{dx}$ of curve. AND Correct gradient of tangent. OR Correct equation of tangent involving expression for $\frac{dy}{dx}$.	Correct solution with correct derivatives.

2018 Question 3d.

(d)	$\frac{dy}{dx} = 2x \cdot \ln x + x^2 . \frac{1}{x}$ $\frac{dy}{dx} = 2x \cdot \ln x + x$ $x = e \Rightarrow \frac{dy}{dx} = 2e \cdot \ln e + e$ $= 3e$ Equation of tangent: $y - y_1 = m(x - x_1)$ $y - e^2 = 3e(x - e)$ $y - e^2 = 3ex - 3e^2$ $y = 3ex - 2e^2$ $(y = 8.155x - 14.778)$	Correct expression for $\frac{dy}{dx}$	Correct solution with correct derivative. Accept any equivalent form.	

2017 Question 1b.

(b)	$\frac{dy}{dx} = \frac{(x+2).2e^{2x} - e^{2x}}{(x+2)^2}$ At $x = 0$ $\frac{dy}{dx} = \frac{2 \times 2 - 1}{4} = \frac{3}{4}$	Correct solution with correct derivative.		

2017 Question 1c.

(c)	$y = 0.5(x-3)^2 + 2$ $\frac{dy}{dx} = 2 \times 0.5 \times (x-3)$ $= x - 3$ At $x = 1$ $\frac{dy}{dx} = -2$ $\therefore$ For normal $\frac{dy}{dx} = \frac{1}{2}$ Through (1, 4) $\therefore$ Eqn of normal $y = \frac{1}{2}x + 3.5$ At point P: $\frac{1}{2}x + 3.5 = 0.5(x-3)^2 + 2$ $x + 7 = (x-3)^2 + 4$ $x + 7 = x^2 - 6x + 9 + 4$ $x^2 - 7x + 6 = 0$ $(x-6)(x-1) = 0$ At point P $x = 6$	Correct expression for $\frac{dy}{dx}$ (i.e. correct derivative).	Correct solution with correct derivative.	

2017 Question 2c.

(c)	$y=\sqrt{x}$ $\frac{dy}{dx}=\frac{1}{2}x^{\frac{-1}{2}}=\frac{1}{2\sqrt{x}}$ At (4, 2) $\frac{dy}{dx}=\frac{1}{2\sqrt{4}}=\frac{1}{4}$ Tangent: $y=\frac{1}{4}x+c$ through (4,2) $2=1+c$ $c=1$ $y=\frac{1}{4}x+1$ $y=0 \Rightarrow 0=\frac{1}{4}x+1$ $x=-4$ Point Q is (−4,0).	Correct derivative.	Correct solution with correct derivative. Accept $x=-4$.	

2016 Question 1d.

(d)	$y=\frac{1}{4}(x-2)^2$ $\frac{dy}{dx}=\frac{1}{2}(x-2)$ At Q (6, 4) $\frac{dy}{dx}=\frac{1}{2}(6-2)=2$ $\therefore$ At P $\frac{dy}{dx}=\frac{-1}{2}$ $\frac{-1}{2}=\frac{1}{2}(x-2)$ $-1=x-2$ $x=1$	At P $\frac{dy}{dx}=\frac{-1}{2}$	Correct solution with correct derivative.	

2016 Question 2b.

(b)	$y=(2x-1)^{\frac{1}{2}}$ $\frac{dy}{dx}=\frac{1}{2}(2x-1)^{\frac{-1}{2}}.2$ $=\frac{1}{\sqrt{2x-1}}$ At $x=5$, $\frac{dy}{dx}=\frac{1}{3}$	Correct solution with correct derivative.		

2016 Question 3b.

(b)	$y = 6x - e^{3x}$ $\frac{dy}{dx} = 6 - 3e^{3x}$ Want $\frac{dy}{dx} = 0$ $3e^{3x} = 6$ $e^{3x} = 2$ $x = \frac{\ln 2}{3} = 0.231$	Correct solution with correct derivative.		

2015 Question 1b.

(b)	$\frac{dy}{dx} = 3(4x - 3x^2)^2(4 - 6x)$ At $x = 1$, $\frac{dy}{dx} = 3 \times 1 \times -2 = -6$	Correct solution with correct derivative.		

2015 Question 1d.

(d)	$f'(x) = \frac{x(x-5) - (x+4)(2x-5)}{x^2(x-5)^2}$ $f'(x) = 0 \Rightarrow x(x-5) - (x+4)(2x-5) = 0$ $x^2 - 5x - (2x^2 + 3x - 20) = 0$ $-x^2 - 8x + 20 = 0$ $x^2 + 8x - 20 = 0$ $(x+10)(x-2) = 0$ $x = -10 \text{ or } +2$	Correct derivative.	Correct solution with correct derivative.	

2015 Question 2b.

(b)	$\frac{dy}{dx} = 1 + \frac{16}{x^2}$ At $x = 4$, $\frac{dy}{dx} = 2$ $\therefore$ Gradient of normal $= \frac{-1}{2}$	Correct solution with correct derivative.		

2015 Question 3a.

(a)	$f'(x) = \frac{5}{2x-3} \times 2 = \frac{10}{2x-3}$ $\frac{10}{2x-3} = 4$ $8x - 12 = 10$ $x = 2.75$	Correct solution with correct derivative.		

2014 Question 1b.

(b)	$\frac{dy}{dx}=2(3x^2-5x)(6x-5)$ At $x=1$, $\frac{dy}{dx}=2\times{}^-2\times1={}^-4$ Gradient of normal $=\frac{1}{4}$ through (1,4)	A correct solution.		

2014 Question 1d.

(d)	$y=\frac{4}{e^{2x-2}}+8x=4e^{-2x+2}+8x$ $\frac{dy}{dx}=-8e^{-2x+2}+8$ Parallel to x-axis $\Rightarrow \frac{dy}{dx}=0$ $8e^{-2x+2}=8$ $e^{-2x+2}=1$ $-2x+2=0$ $x=1$	A correct expression for $\frac{dy}{dx}$.	A correct solution.	

2013 Question 2b.

(b)	$\frac{dy}{dx}=3(x^3-2x)^2\cdot(3x^2-2)$ At $x=1$, $\frac{dy}{dx}=3\cdot(-1)^2\cdot1=3$ At $x=1$, $y=-1$ $y+1=3(x-1)$ $y=3x-4$	Correct solution with correct derivative shown.		