Differentiation Collated Past Answers - Differentiation

2023 Question 1a.

ONE (a)	$\frac{dy}{dx}=\frac{1}{2}(3x-2)^{-\frac{1}{2}}\times 3=\frac{3}{2}(3x-2)^{-\frac{1}{2}}$	• Correct derivative.		

2023 Question 2a.

TWO (a)	$f'(x)=\frac{(\cos x)(2x)-\left(x^2\right)(-\sin x)}{\cos^2 x}$ $=\frac{2x\cos x+x^2\sin x}{\cos^2 x}$ $=\frac{x(2\cos x+x\sin x)}{\cos^2 x}$	• Correct derivative.		

2023 Question 2b.

(b)	$\frac{dy}{dx}=-2\operatorname{cosec}^2(2x)$ When $x=\frac{\pi}{12}$ $\frac{dy}{dx}=\frac{-2}{\sin^2\left(\frac{\pi}{6}\right)}$ $=-8$	• Correct derivative. AND Correct gradient of -8 found.		

2023 Question 3a.

THREE (a)	$\frac{dy}{dx}=\frac{2x-4x^3}{x^2-x^4+1}$	• Correct derivative.		

2022 Question 1a.

ONE (a)	$\frac{dy}{dx}=2\ln x\cdot\frac{1}{x}$	Correct derivative		

2022 Question 2a.

TWO (a)	$f'(x)=4(5x-3)\cos 4x+5\sin 4x$	Correct derivative.		

2022 Question 3a.

THREE (a)	$\frac{dy}{dx} = e^{4\sqrt{x}}.2x^{\frac{-1}{2}}$	Correct derivative		

2021 Question 1a.

ONE (a)	$\frac{dy}{dx} = 3e^{3x}\sin(2x) + e^{3x}\cos(2x).2$	Correct derivative.		

2021 Question 2a.

TWO (a)	$\frac{dy}{dx} = 5(1-x^2)^4 \times (-2x)$	Correct derivative.		

2021 Question 3a.

THREE (a)	$\frac{dy}{dx} = \frac{(x^2+1)(-\text{cosec}^2 x) - (\cot x)(2x)}{(x^2+1)^2}$	Correct derivative.		

2020 Question 1a.

ONE (a)	$\frac{dy}{dx} = 5(3x-x^2)^4 \cdot (3-2x)$	Correct derivative.		

2020 Question 2a.

TWO (a)	$\frac{dy}{dx} = \frac{x^3 \cdot \sec^2 x - 3x^2 \tan x}{x^6}$	Correct derivative		

2020 Question 3a.

THREE (a)	$\frac{dy}{dx} = 3 \times \frac{1}{x^2-1} \times 2x$ $= \frac{6x}{x^2-1}$	Correct derivative.		

2020 Question 3e.

(e)	$\frac{dy}{dx} = (3x+2)e^{-2x}.(-2)+3e^{-2x}$ $= e^{-2x}\left[-2(3x+2)+3\right]$ $= e^{-2x}(-6x-1)$ $\frac{d^2y}{dx^2} = -6e^{-2x} - 2e^{-2x}(-6x-1)$ $= e^{-2x}\left[-6-2(-6x-1)\right]$ $= e^{-2x}(-6+12x+2)$ $= e^{-2x}(12x-4)$ $= 4e^{-2x}(3x-1)$ EITHER $\frac{d^2y}{dx^2}+4\frac{dy}{dx}+4y=0$ LHS $= 4e^{-2x}(3x-1)+4e^{-2x}(-6x-1)+4e^{-2x}(3x+2)$ $= 4e^{-2x}\left[3x-1-6x-1+3x+2\right]$ $= 0$ = RHS as required OR LHS $= e^{-2x}(12x-4)+4e^{-2x}(-6x-1)+4e^{-2x}(3x+2)$ $= e^{-2x}\left[12x-4+4(-6x-1)+4(3x+2)\right]$ $= e^{-2x}\left[12x-4+24x-4+12x+8\right]$ $= 0$ = RHS as required	Correct expression for $\frac{dy}{dx}$.	Correct expression for $\frac{d^2y}{dx^2}$.	Correct solution with correct derivatives.

2019 Question 1a.

(a)	$\frac{dy}{dx} = \frac{1}{2}(3x^2-1)^{\frac{-1}{2}}.6x$ $= \frac{3x}{\sqrt{3x^2-1}}$	Correct derivative. Anything equivalent.		

2019 Question 1b.

(b)	$f'(t) = \frac{15}{3t-1}$ $f'(4) = \frac{15}{11}$ or 1.36	Correct solution with correct derivative.		

2019 Question 2a.

(a)	$\frac{dy}{dx} = 4(2x-5)^3.2$ $\frac{dy}{dx} = 8(2x-5)^3$	Correct derivative.		

2019 Question 2b.

(b)	$\frac{dy}{dx} = 2\sec^2 2x$ $= \frac{2}{\cos^2 2x}$ At $x = \frac{\pi}{6}, \frac{dy}{dx} = \frac{2}{\cos^2 \frac{\pi}{3}} = 8$	Correct solution with correct derivative.		

2019 Question 3a.

(a)	$-4\sin^{-2} x\cos x$ OR $-4\operatorname{cosec} x\cot x$	Correct derivative.		

2018 Question 1a.

(a)	$6x^2 - 15(x^3+2)^{-4}.3x^2$	Correct derivative.		

2018 Question 1b.

(b)	$f'(x) = -9\sin 3x$ $f''(x) = -27\cos 3x$ $9f(x) + f''(x)$ $= 9(3\cos 3x) - 27\cos 3x$ $= 27\cos 3x - 27\cos 3x$ $= 0$	Correct proof with correct first and second derivatives.		

2018 Question 1c.

(c)	$y = \ln\lvert\sin^2 x\rvert$ $\frac{dy}{dx} = \frac{2\sin x\cos x}{\sin^2 x}$ $= \frac{2\cos x}{\sin x}$ OR $y = \ln\lvert\sin^2 x\rvert$ $= 2\ln\lvert\sin x\rvert$ $\frac{dy}{dx} = \frac{2\cos x}{\sin x}$ etc When $x = \frac{\pi}{6}$, $\frac{dy}{dx} = \frac{2\cos\frac{\pi}{6}}{\sin\frac{\pi}{6}}$ $= 2\sqrt{3}$ $(= 3.4641)$	Correct expression for $\frac{dy}{dx}$	Correct solution with correct expression for $\frac{dy}{dx}$	

2018 Question 2a.

(a)	$\frac{3}{2}x^{\frac{-1}{2}} - 5\text{cosec}5x\cot 5x$	Correct derivative.		

2018 Question 2b.

(b)	$v(t)$ $\frac{6t + 3}{3t^2 + 3t + 1}$ $v(2)$ $\frac{15}{19}$ or $0.789\ \text{m s}^{-1}$	Correct solution with correct derivative.		

2018 Question 3a.

(a)	$\frac{\left(x^2+1\right).2e^{2x} - e^{2x}.2x}{\left(x^2+1\right)^2}$	Correct derivative.		

2017 Question 1a.

(a)	$\frac{1}{2}x^{\frac{-1}{2}} + 2\sec^2(2x)$	Correct solution.		

2017 Question 2a.

(a)	$\frac{dy}{dx} = 10\left(x^2 - 4x\right)^4.(2x-4)$	Correct derivative.		

2017 Question 3a.

(a)	$\frac{dy}{dx} = x.\frac{3}{3x-1} + \ln(3x-1)$	Correct derivative.		

2017 Question 3b.

(b)	$y = x^{-1} - x^{-2}$ $\frac{dy}{dx} = -x^{-2} + 2x^{-3}$ $= \frac{-1}{x^2} + \frac{2}{x^3}$ At $x = 2$ $\frac{dy}{dx} = \frac{-1}{4} + \frac{2}{8} = 0$	Correct solution with correct derivative.		

2017 Question 3e.

(e)	(i) $\frac{dy}{dx} = e^x.\cos kx + e^x(-k\sin kx)$ $= e^x(\cos kx - k\sin kx)$ $\frac{d^2y}{dx^2} = e^x(\cos kx - k\sin kx)$ $+e^x(-k\sin kx - k^2\cos kx)$ $= e^x(\cos kx - 2k\sin kx - k^2\cos kx)$ (ii) $\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + 2y = 0.$ $\Rightarrow e^x(\cos kx - 2k\sin kx - k^2\cos kx)$ $-2e^x(\cos kx - k\sin kx) + 2e^x\cos kx = 0$ $\Rightarrow e^x(\cos kx - k^2\cos kx) = 0$ $e^x\cos kx(1-k^2) = 0$ $k = \pm 1$	Correct expression for $\frac{dy}{dx}$	Correct expression for $\frac{d^2y}{dx^2}$	Correct solution with correct derivatives.

2016 Question 1a.

(a)	$\frac{dy}{dx} = 1 + x^{-2} - 2x^{-3}$	Correct solution		

2016 Question 2a.

(a)	$f'(x) = \ln(3x-1) + x \cdot \frac{3}{3x-1}$	Correct derivative		

2016 Question 3a.

(a)	$f'(x) = \frac{1}{4}(3x+2)^{\frac{-3}{4}} \cdot 3$	Correct derivative.		

2016 Question 3d.

(d)	$y = \frac{e^x}{\sin x}$ $\frac{dy}{dx} = \frac{\sin x \cdot e^x - e^x.\cos x}{\sin^2 x}$ $= \frac{\sin x \cdot e^x}{\sin^2 x} - \frac{e^x.\cos x}{\sin^2 x}$ $= \frac{e^x}{\sin x} - \frac{e^x}{\sin x} \cdot \frac{\cos x}{\sin x}$ $= y - y \cdot \cot x$ $= y(1 - \cot x)$	Correct expression for $\frac{dy}{dx}$.	Correct proof with correct derivative.	

2015 Question 1a.

(a)	$30\sec^2(5x)$	A correct expression for the derivative.		

2015 Question 2a.

(a)	$\frac{1}{5}(x - 3x^2)^{\frac{-4}{5}} \cdot (1 - 6x)$	A correct expression for the derivative.		

2015 Question 3d.

(d)(i)	$\frac{dx}{dt} = -Ak\sin kt + Bk\cos kt$ $\frac{d^2x}{dt^2} = -Ak^2\cos kt - Bk^2\sin kt$ $= -k^2(A\cos kt + B\sin kt)$ $= -k^2x$	Correct $\frac{dx}{dt}$ Or $\frac{d^2x}{dt^2}$ Consistent with	Parts (i) and (ii) both correct.	
(ii)	$x(0) = 0 \Rightarrow A\cos 0 + B\sin 0 = 0$ $A = 0$ $v(0) = 2k \Rightarrow 2k = -Ak\sin(0) + Bk\cos(0)$ $B = 2$	incorrect $\frac{dx}{dt}$		

2014 Question 1a.

(a)	$-15\sin(3x)$	A correct expression for the derivative.		

2014 Question 2a.

(a)	$f'(x) = \frac{(2x-1)4e^{4x} - e^{4x}.2}{(2x-1)^2}$	A correct expression for the derivative.		

2014 Question 2b.

(b)	$y=8\ln(3x-2)$ $\frac{dy}{dx}=\frac{24}{(3x-2)}$ At $x=2$ $\frac{dy}{dx}=6$	A correct solution.		

2014 Question 3a.

(a)	$y=\left(\sqrt[3]{x^2+4x}\right)^2=(x^2+4x)^{\frac{2}{3}}$ $\frac{dy}{dx}=\frac{2}{3}(x^2+4x)^{\frac{-1}{3}}(2x+4)$	A correct expression for the derivative.		

2013 Question 1a.

(a)	$\frac{dy}{dx}=\sec^2\left(x^2+1\right)\cdot 2x$	Correct derivative.		

2013 Question 1b.

(b)	$\frac{dy}{dx}=\frac{3-e^x}{3x-e^x}$ or no tangent exists At $x=0$ gradient $=-2$	Correct solution with correct derivative shown.		

2013 Question 2a.

(a)	$\frac{dy}{dx}=\frac{1}{3}\left(\pi-x^2\right)^{\frac{-2}{3}}\cdot -2x$ or $\frac{dy}{dx}=\frac{-2x}{3\left(\pi-x^2\right)^{\frac{2}{3}}}$	Correct derivative.		

2013 Question 3a.

(a)	$\frac{dy}{dx}=\frac{x^2\cdot\cos 2x\cdot 2-2x\sin 2x}{x^4}$	Correct derivative.		