Name:_____ Teacher: _____

Year 12 Probability Simulations
AS 2.13
Internal 2 Credits
Student Booklet

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Investigate a situation using a simulation.	Investigate a situation using a simulation, with justification.	Investigate a situation using a simulation, with statistical insight.

Achievement	Achievement with Merit	Achievement with Excellence
Students will show evidence of investigating the situation using each component of the simulation process. This will mean: designing the simulation for the given situation, identifying the tools to be used, defining what is a trial and the number of trials, determining the data recording methods; carrying out the simulation and recording the outcomes; selecting and using appropriate displays and measures; and communicating findings in a conclusion. In their report the student should show evidence that they have: • Designed the simulation for the situation given. They have specified the tools to be used, what a trial is, the number of trials and the data recording method. • Conducted the initial simulation and recorded the outcomes. • Selected and used appropriate displays and measures. • Communicated findings clearly, making a conclusion relating to the simulation.	Students will show evidence of investigating the situation using each component of the simulation process, linking this to the context, explaining the relevant design considerations made in the design and supporting findings. This will mean: designing the simulation for the given situation, identifying the tools to be used, defining what is a trial and the number of trials, determining the data recording methods; carrying out the simulation and recording the outcomes; selecting and using appropriate displays and measures; and communicating findings in a conclusion. In their report the student should show evidence that they have: • Designed the simulation for the situation given. They have described in detail the tools to be used, what a trial is, the number of trials and the data recording method. They have identified at least one assumption in designing their simulation. • Conducted the initial simulation and recorded the results and repeated the simulation. • Selected and used appropriate displays and measures. • Communicated findings clearly, and linked the recommendation to the results of the simulations.	Students will show evidence of investigating the situation using each component of the simulation process, integrating statistical and contextual knowledge throughout the process. This will mean: designing the simulation for the given situation, identifying the tools to be used, defining what is a trial and the number of trials, determining the data recording methods; carrying out the simulation and recording the outcomes; selecting and using appropriate displays and measures; and communicating findings in a conclusion. In their report the student should show evidence that they have: • Designed the simulation for the situation given. They have described in detail the tools to be used, what a trial is, the number of trials and the data recording method. They have identified at least two assumptions in designing their simulation. • Conducted the initial simulation and recorded the results and repeated the simulation • Selected and used appropriate displays and measures. They have discussed the overall aspects of the distribution or simulation. • Communicated findings clearly, and linked the recommendation clearly to the results of the both simulations. They have discussed more than one aspect of the recommendation with respect to the context of the simulation in depth.

Introduction

Simulations are used to solve probability problems when it is difficult to calculate the answer theoretically. You may be asked to calculate

- The long run frequency of an event happening
- The average number of times needed to carry out a 'trial' until a certain condition is met

Simulation Tools

When designing a simulation a tool must be chosen that matches the situation. Possible tools include

- Coins, Dice, Spinners
- Random number tables
- Random numbers on the calculator
- Random number generator on a spreadsheet

Random numbers on the calculator

The RAN# button produces random numbers from 0 to 0.999 999 999 9. It is possible to use the random numbers generated directly (see your teacher) or you can tailor the numbers to the specific problem.

TTRCA

Towards achievement in Achievement Standard 2.13:
Investigate a situation using a simulation.

To describe a simulation you should cover the following **five** aspects:

Tool: State which tool you will use.
State how the tool will be used to model the situation.

Trial: State what one trial consists of.
State how you will recognise success.
State how many trials you will do.

Results: Draw up a table to present your results. Use clear headings.
Generally you should do at least thirty trials.
Include any random numbers you generate.

Calculation: Carry out the calculation required to solve the problem

Answer: Answer the question (read it again first)

Write-on exercise 1 The Three Child Family

Problem: What is the probability that a 3-child family will contain exactly two boys and one girl? You may assume that a boy or girl is equally likely to be born.

Tool:

_____

_____

Trial:

_____

_____

Results (for example if a 2 represents a boy; you are to do your own trials):

Trial	Outcome of trial	Two boys exactly	
1	122	yes	
2	111	no	
3			

Write your results in your book.

Calculation:

_____

_____

_____

Answer: Use the results of your simulation to answer these questions:

In 250 3-child families how many would you expect to have

a) Exactly 2 boys

_____

_____

_____

b) All girls

_____

_____

_____

We could answer the question *"What is the probability that a 3-child family will contain exactly two boys and one girl?"* more easily by drawing a probability tree and working out the theoretical probability. From now on we will only do simulations of situations where we can't easily work out the theoretical probability.

Write on Exercise 2

Breakfast Gifts

Problem: A cereal manufacturer includes a gift in each packet of cereal. There are four different gifts. What is the expected number of packets of cereal you must buy to collect all four different gifts? Assumption: the gifts are randomly distributed in the boxes of cereal and each gift is equally likely to be in a box.

Tool:

_____

_____

_____

Trial: One trial consists of

- Generating random numbers on the calculator until at least one of each number 1 to 4 turns up
- The number of packets of cereal required is recorded
- 30 trials will be done

Results: The results will be tabulated as follows (for example):

Trial	Outcome of trial	Number of packets
1	21113214	8
2	334114432	9

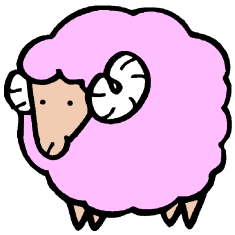
Write your table of results in your book.

Calculation: Estimate of the number of cereal packets needed:

average number of packets = $\frac{\text{total the number of packets for the 30 trials}}{30}$ =

Answer:

Exercise 4 — A Woolly Task



Farmer McDonald wants to know the average number of lambs produced by his special flock of 720 breeding seasons. Assume that each ewe produces either a single lamb or twin lambs. From records of past breeding seasons Farmer McDonalds knows that the probability of breeding twins is 1/6.

1. Design a model to simulate the lamb production of a ewe for 5 successive breeding seasons.
   Describe the steps you will take in sufficient detail so that the farmer can repeat it without your help.
2. Carry out simulation by conducting 30 trials and recording your data.
3. Use the results of your simulation to find the mean number of lambs produced by an ewe over the four seasons.
4. Use the results of your simulation to estimate the number of ewes in this flock that have at least two sets of twins given that they have twins in the first season.
5. Farmer McDonald is kind to his sheep, so an ewe which has produced two sets of twins will no longer be used for breeding. Use the results of your simulation to estimate the number of ewes (in the flock of 720) that will still be used for breeding after 3 years.
6. What are the limitations of the model you have chosen in predicting the number of lambs produced by the McDonald flock over several years? Give at least two limitations.

Exercise 5 — Bob's ties

Problem: Bob has 7 different ties. He works a five day week in the office of the Inland Revenue. Design and carry out a simulation to calculate the probability that Bob wears the same tie more than once in a five day week. You may assume that he chooses a tie randomly each morning.

Exercise 6 — Oil Strike

Problem: Suppose that the probability that an exploratory oil well will strike oil is 0.2 and that each exploratory well costs 5 million dollars to drill. How many of the next 20 wells established will have cost at least 25 million dollars for their exploration?

Exercise 7 — Boys wanted!

Problem: In many countries around the world, couples look to a son to take care of them in their old age. For this reason they keep having children until they give birth to a son. This can lead to over-population. However, suppose a government permits people to continue having children until they have exactly one son. Design and carry out a simulation to answer the following questions:

a) What is the average number of children per family in these circumstances?
b) What is the probability that a couple will have three children without having a boy?

Exercise 8 An Infectious Disease

The speed of an infectious disease can be modelled as follows. Suppose that an infectious disease has a one-day infection period, and after that a person is immune. Six people live on an otherwise deserted island. One person catches the disease and randomly visits one other person at random during the next day (his infection period). The process continues, with one visit per day, until an infectious person visits an immune person and the disease dies out.

Design and carry out a simulation to solve the problem. Write down all 5 steps (TTRCA).

Construct at least 30 trials of the simulated epidemic. Use the simulation results to answer the following questions.

(a) What is the average number of people who get disease in your simulated epidemic?
(b) What is the probability that more than 3 people will get the disease?
(c) What is the probability that all six people get the disease?
(d) What are the limitations of the model you have chosen in predicting the number of people catching the disease? Give at least two limitations.