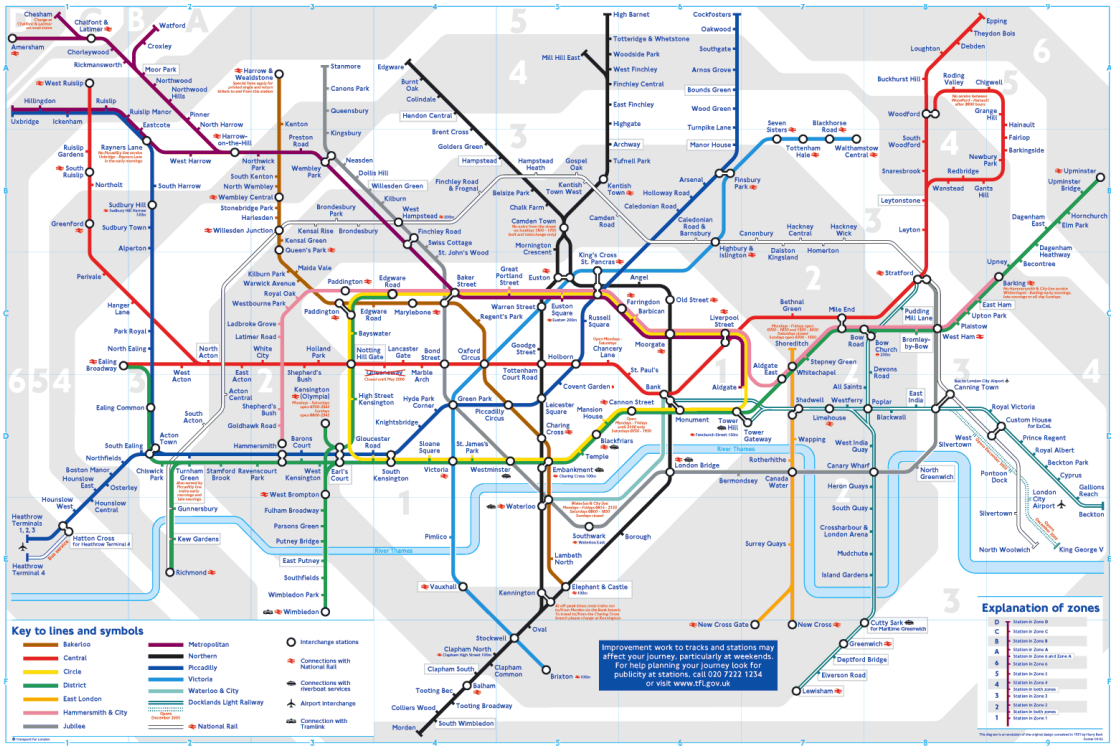
2.5
Networks


Key to lines and symbols
Bakerloo
Central
Circle
District
East London
Hammersmith & City
Jubilee
Metropolitan
Northern
Piccadilly
Victoria
Waterloo & City
Docklands Light Railway
National Rail
Interchange stations
Connections with National Rail
Connections with riverboat services
Airport interchange
Connection with Tramlink
Improvement work to tracks and stations may affect your journey, particularly at weekends. For help planning your journey look for publicity at stations, call 020 7222 1234 or visit www.tfl.gov.uk
Explanation of zones

Networks are an integral part of doing business
They are used in planning to help find things like the shortest time, distance, smallest cost, largest profit of a project .
For example when a courier drive has to deliver parcels the further he has to drive the longer it takes and the more it costs; if he can save time and distance he will save money.

In this topic there are 3 important concepts to master

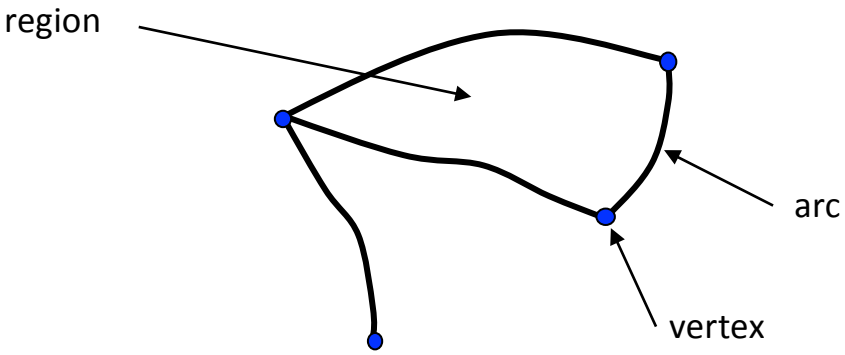
1. Traversability
2. Shortest Path
3. Minimum Spanning Tree

Where you see this bird goto the Padlet to view a **how to** video
http://padlet.com/rheadifen/Networks

Table of Contents

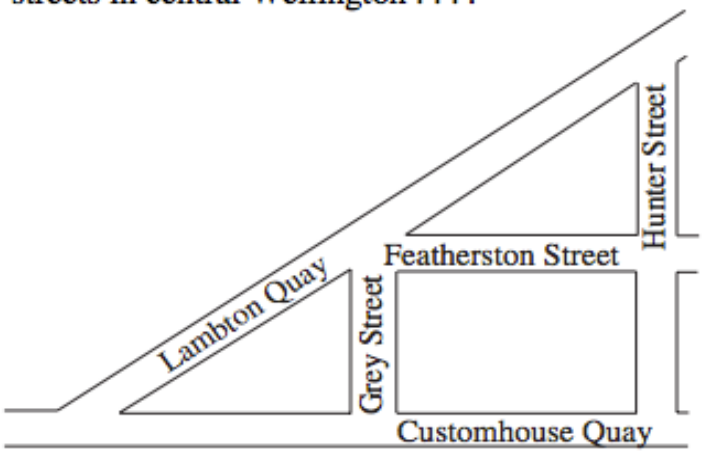
Arcs , Vertices, and Regions.

Networks are made up of arcs, vertices & regions

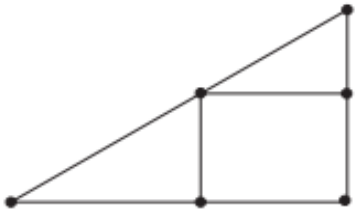


This network has	4 vertices 4 arcs 2 regions	Vertices are also called **Nodes** Arcs are also called edges The area outside the network is also a region

This is a map of a few streets in central Wellington . . . .



. . . and here is a network of this map.



The streets are represented by the arcs and the intersections by the nodes.

Odd & Even

A vertex can be **odd** or **even**.

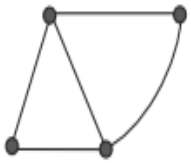
This is decided by the number of arcs connected to it.

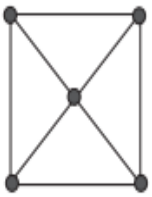
A vertex with an even number of arcs extending from it icalled an even node

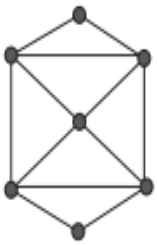
A vertex with an odd number of arcs extending from it icalled an odd node

Work out which nodes are odd & which nodes are even in each of the 4 networks below. Complete the table

a 

b 

c 

d 

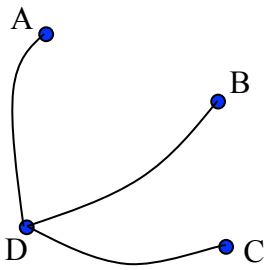
Network	Number of **odd** nodes	Number of **even** nodes
a		
b		
c		
d		

Traversable Networks

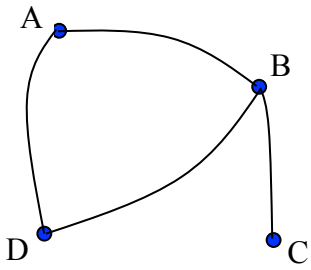
If you can trace over all the arcs (lines) in a network…

- without lifting your pen off the paper
- without drawing any line twice

…then it is traversable.



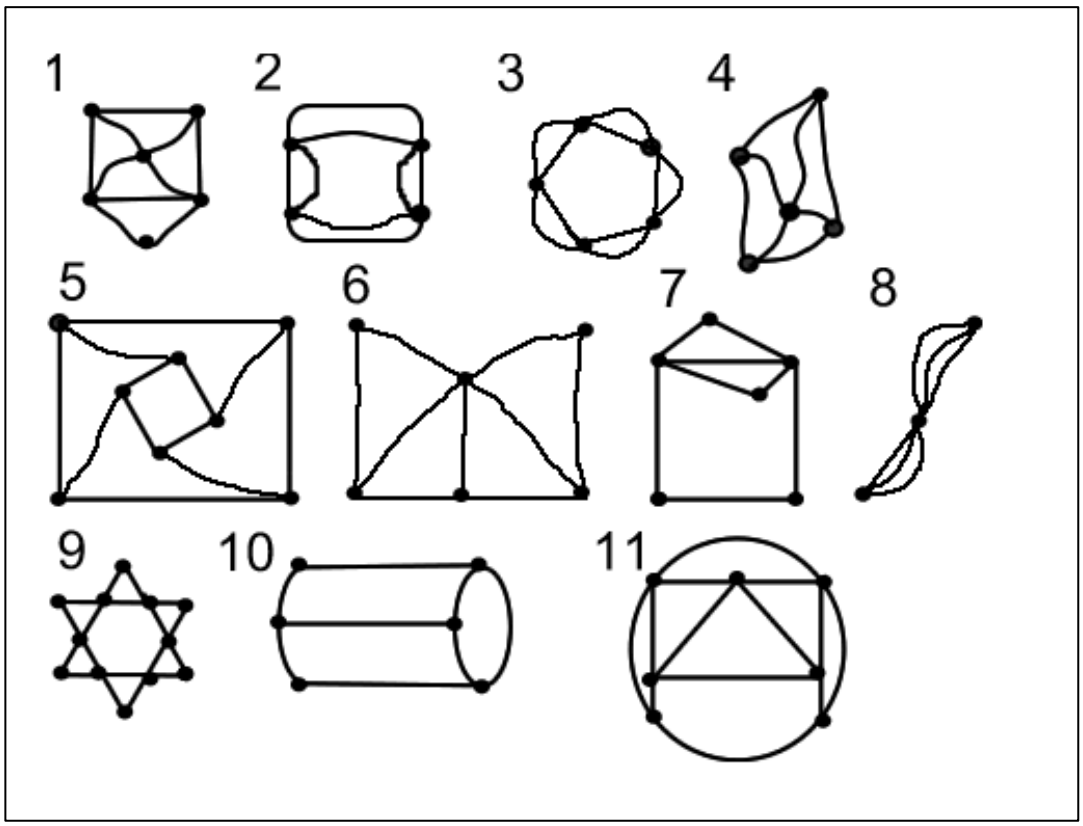
Not Traversible



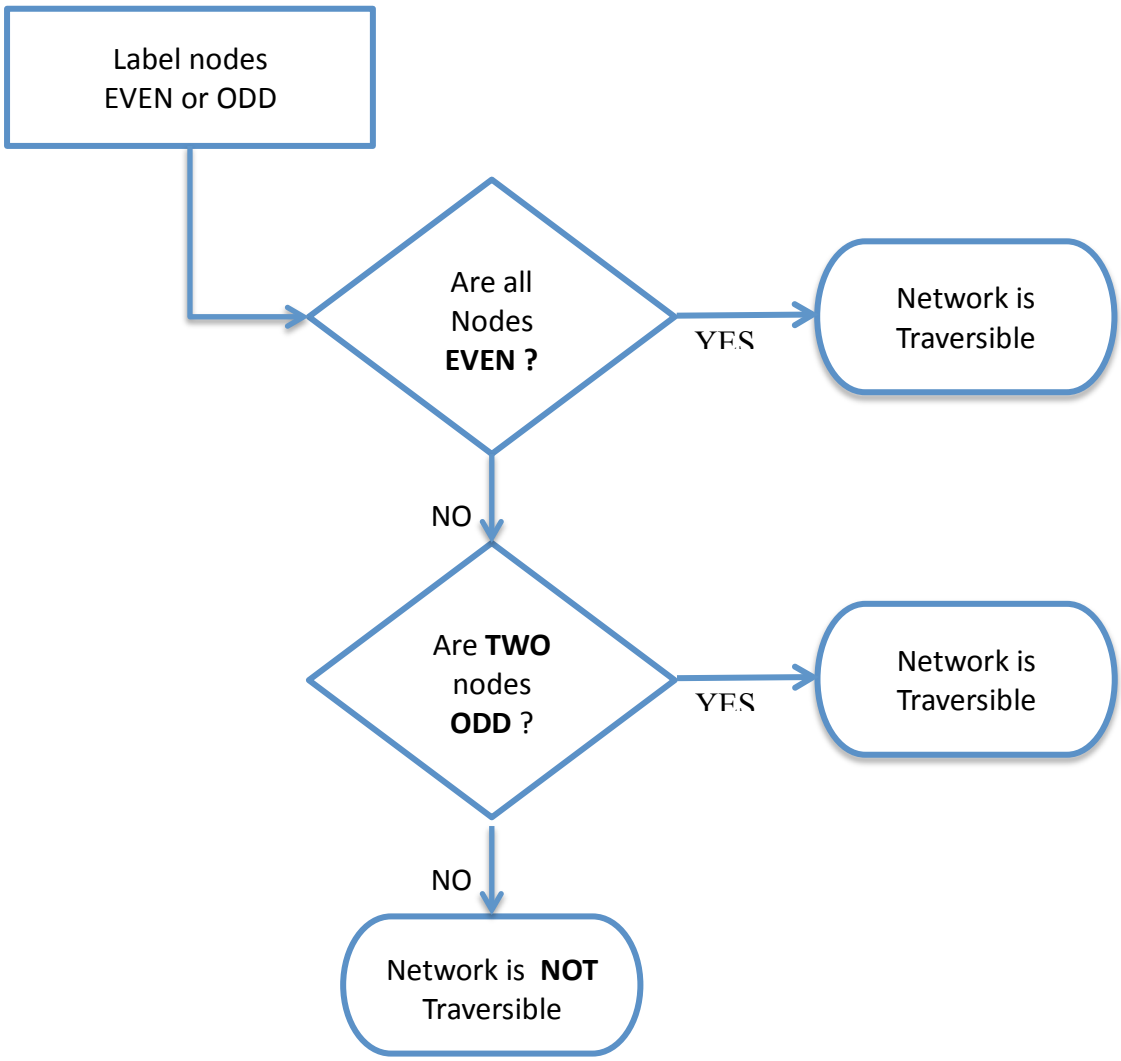
Traversible

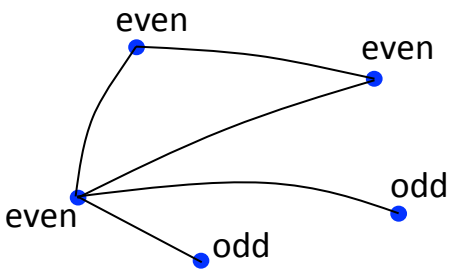
e.g C – B – A – D – B

Decide which of these are traversible

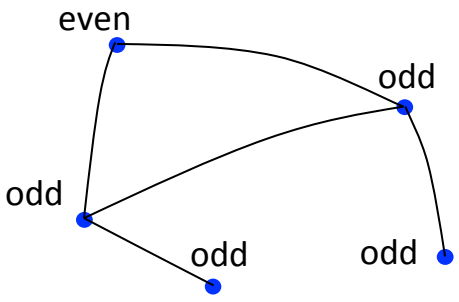


How to check to see is a network is traversable



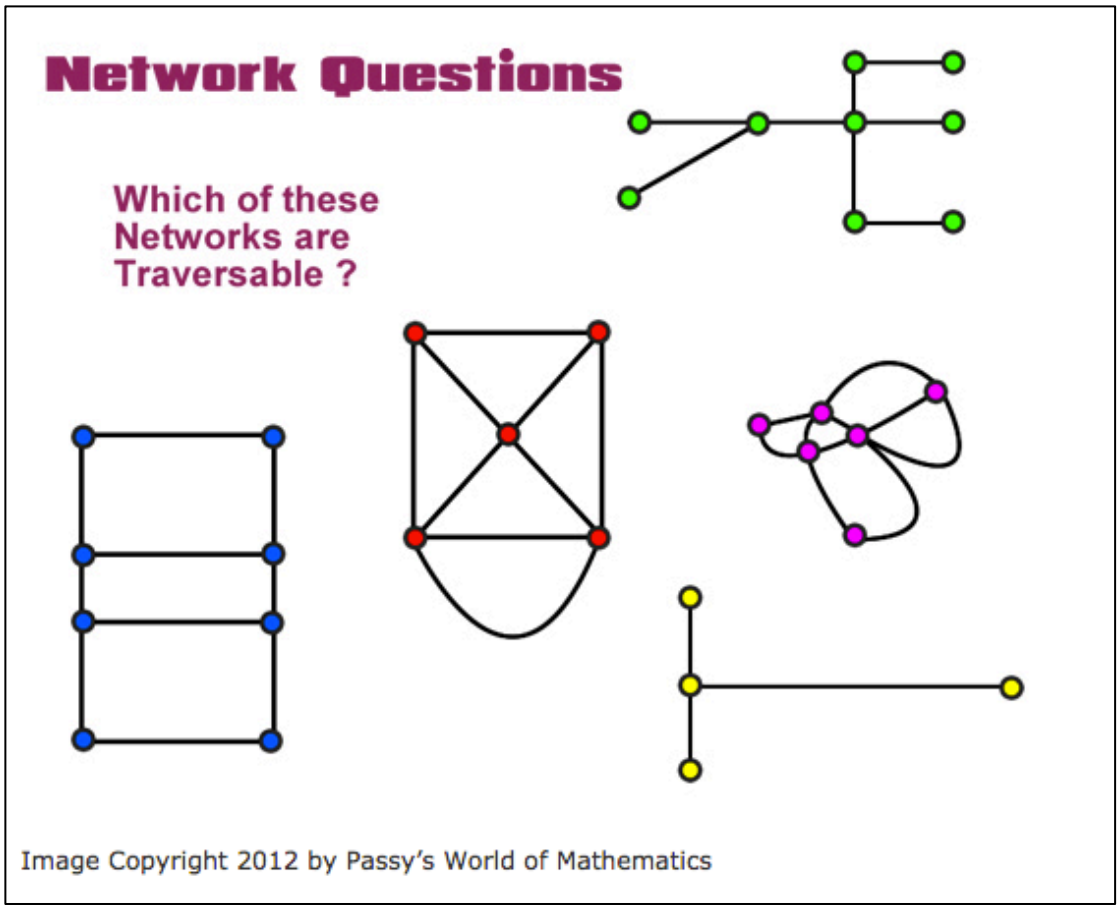


2 odd nodes ∴ traversible

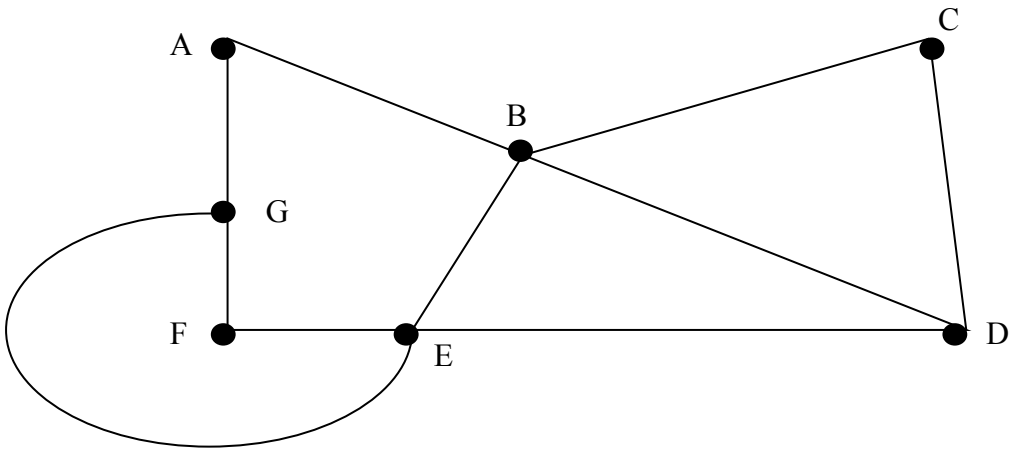


4 odd nodes ∴ not traversible

A network is traversable if all nodes are even or exactly 2 are odd



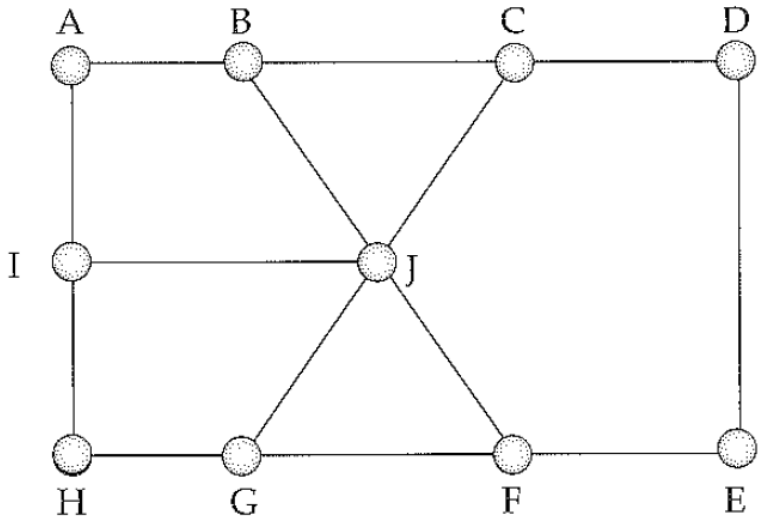
Here is a network for a courier drivers daily route.

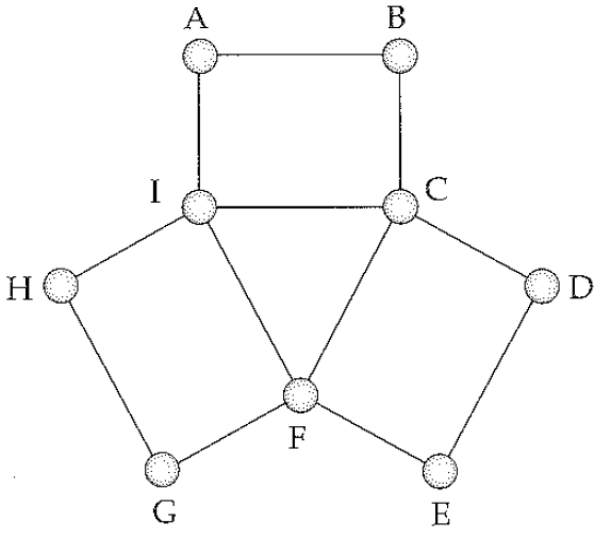


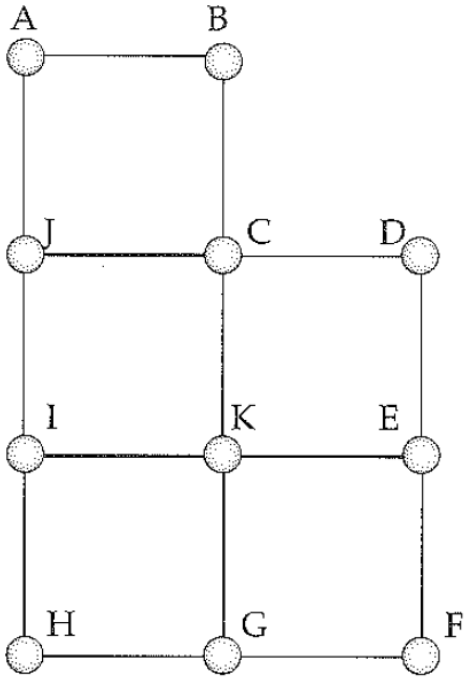
The driver wants to minimise the distance he has to cover. Is it possible for him to travel along each road only once? Justify your answer.

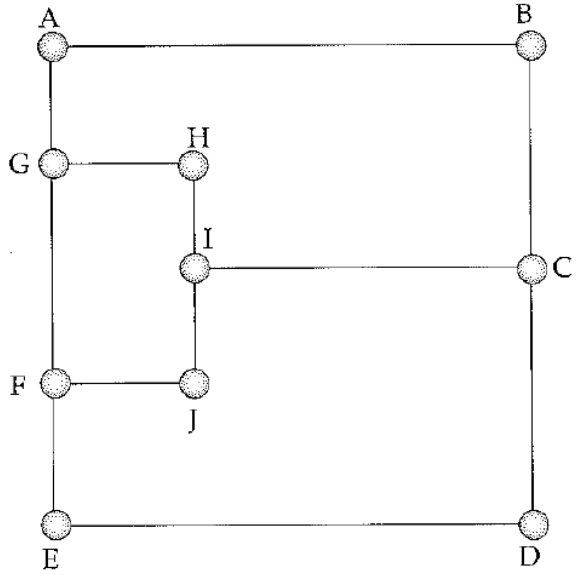
Describe a possible route that he can follow to achieve this (if it is possible).

Are these networks traversable ?









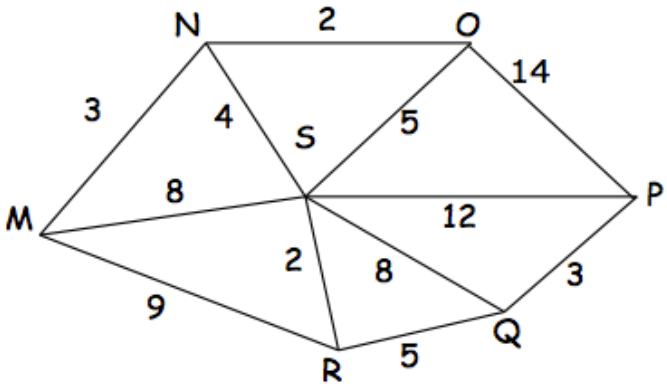
Fom Nulake P14 &15

Shortest Path

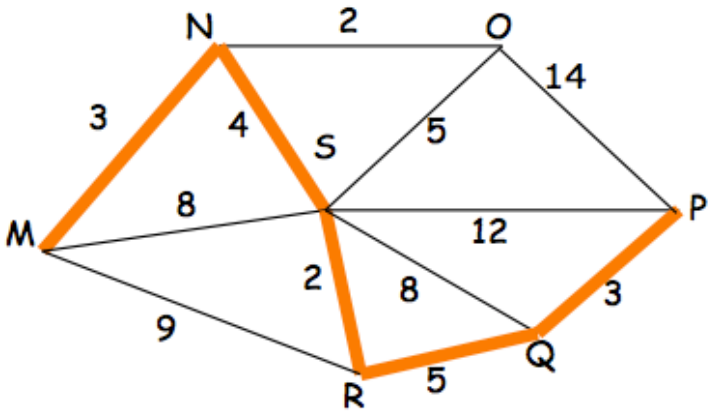
When we look for the shortest path in a network we are looking for the shortest way to get from one point (node) to another.
The shortest path does not usually include all nodes on the network

It is important to check all routes as the shortest path is not always the most obvious route on a network.

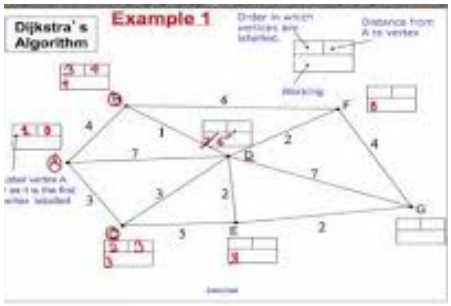
EXAMPLE:
This network shows the distances between towns on a courier route
Distances are measured in km.



Find the shortest path between towns M and P.
Write down the shortest Path and calculate the distance



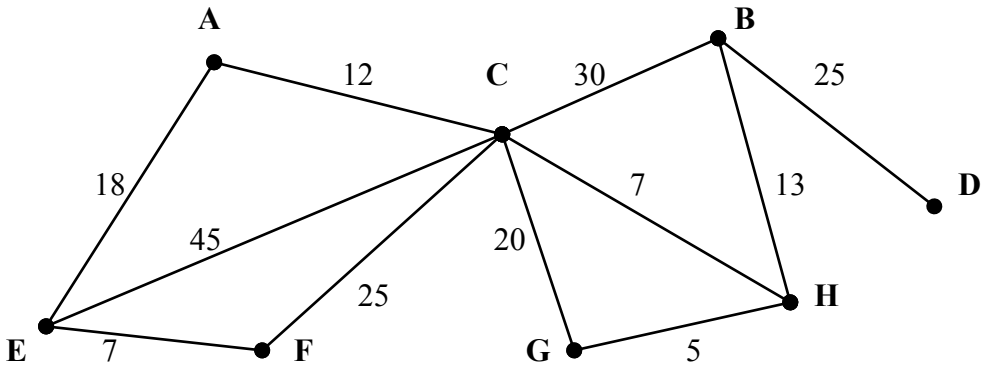
M →N → S → R→ Q → P is the shortest from M to P
The shortest path is 17 km (3+4+2+5 +3)



For small networks the shortest path can be found by simply checking different paths.

For much larger networks we need to be more systematic. Dijkstra's algorithm is often used.
Goto the Padlet page and watch the video to learn how to apply Dijkstra's algorithm

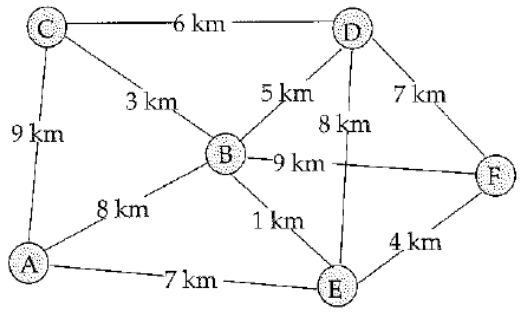
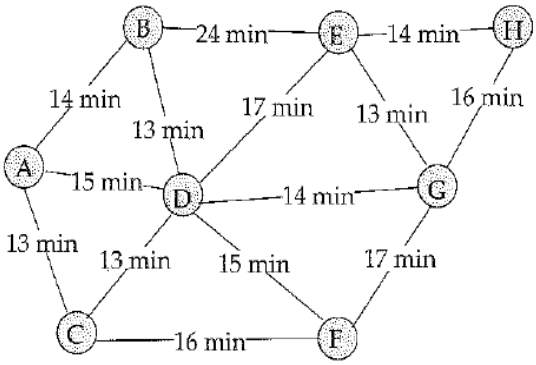
Try This:
An Internet Service Provider (ISP) has a series of towns connected up to their network. Each town is represented by a letter and the numbers are the times (in milliseconds) for data to be transferred between towns.



What is the quickest time that data can be transmitted from Town D to Town E?

_____milliseconds

Highlight the shortest path from Town D to Town E on the diagram above.

Find the shortest path from A to F *Calculate the distance*	*Find the shortest path from A to H* *Calculate the time*
	

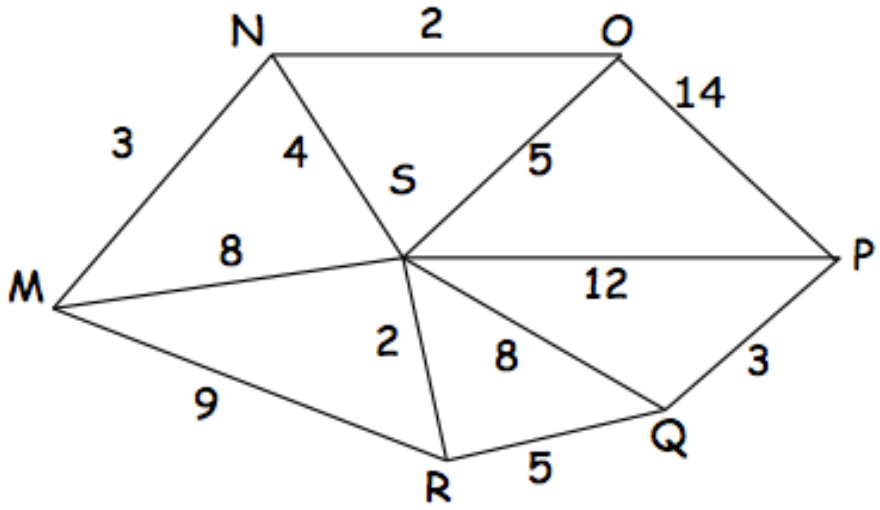
From Nulake P 28 & 29

Minimal Spanning Trees

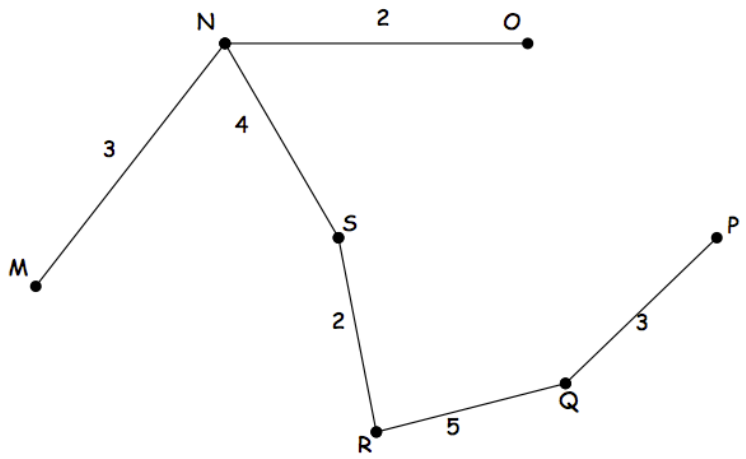
When we find the minimum spanning tree we are looking for the way to connect **all nodes** in the network with the minimum weight.

The weight is the number on the arc. The weight can represent distance, time, cost or some other value.

EXAMPLE
Find the mimimum amount of pipe necessary to connect all of the towns drawn in the network below ie.draw the minimal spanning tree.
Distances are measured in km.



The **minimum spanning tree** for this network is

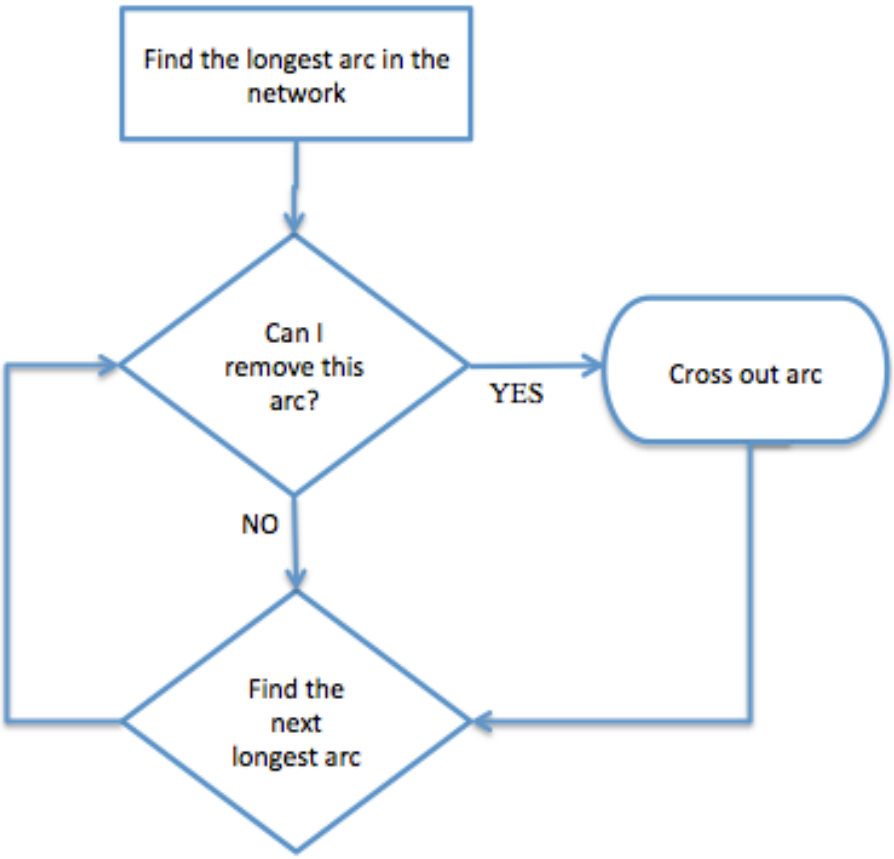


The **minimum length of pipe** needed is 3+4+2+5+3+2 = 19 km

The Reverse Delete Algorithm

The reverse delete algorithm is the method we use to find the minimum spanning tree.

We start with the longest arc and check to see that if we removed it , could we still get to the nodes at each end.
We keep doing this until there are no more arcs to remove.
What is left is the minimum spanning tree (see page 9)

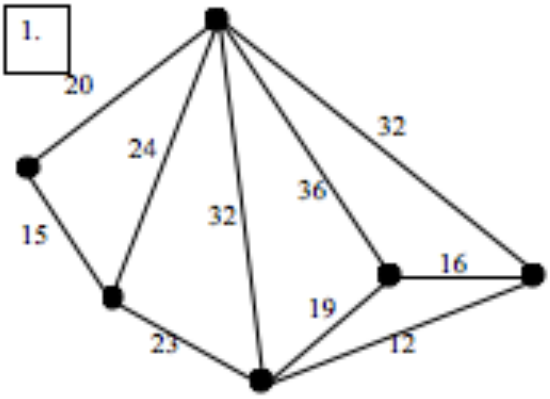


The difference between the minimum spanning tree and the shortest path.

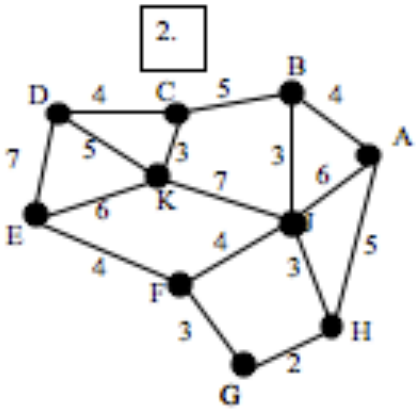
The minimum spanninng tree includes all nodes
The shortes path includes only the nodes necessary to get from A to B

Find the minimal spanning tree and the total minimum cost for the following networks:

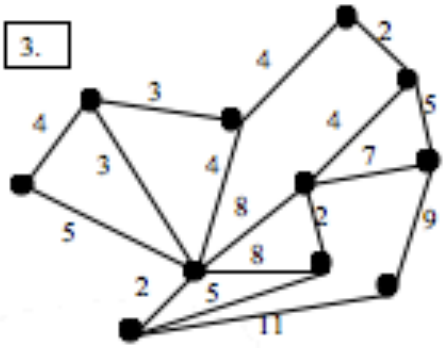
1.



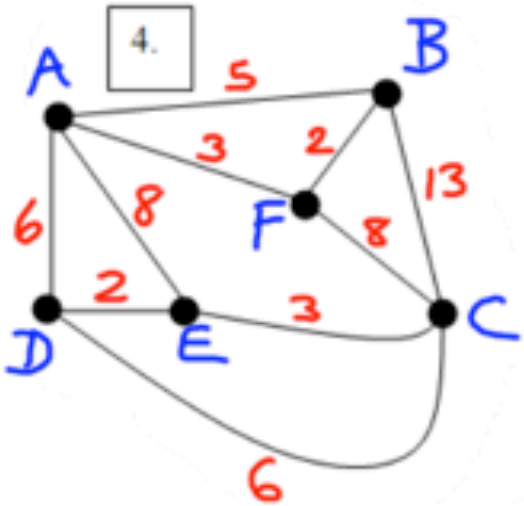
2.



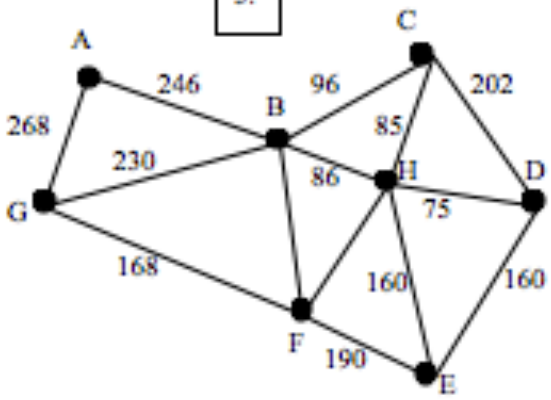
3.



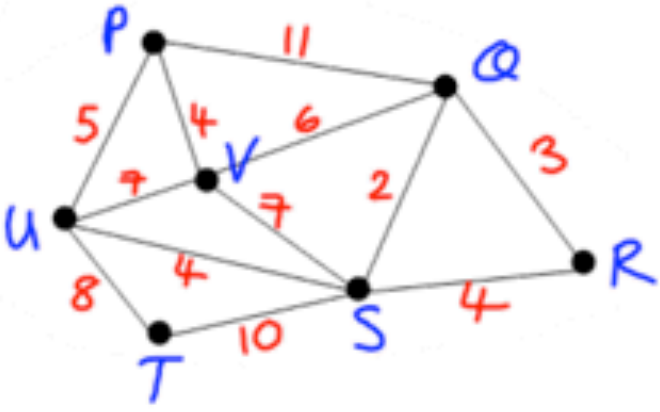
4.



5.



6.

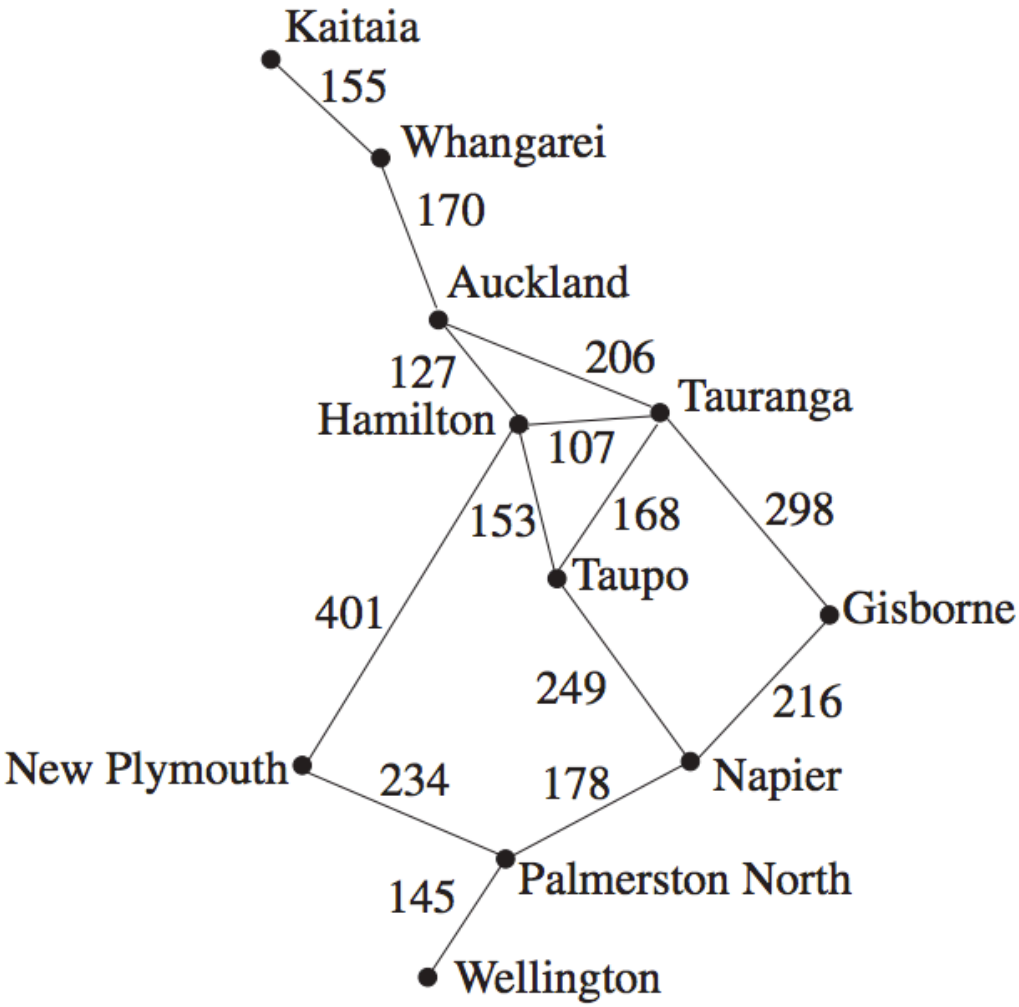


Brilliant

The telecommunications company, Brilliant, wants to link its customers with a *miracle thread* which is embedded in the bitumen in roads.

Below is a map of the centres where the customers are situated and the approximate distances (in kilometres) between the centres.

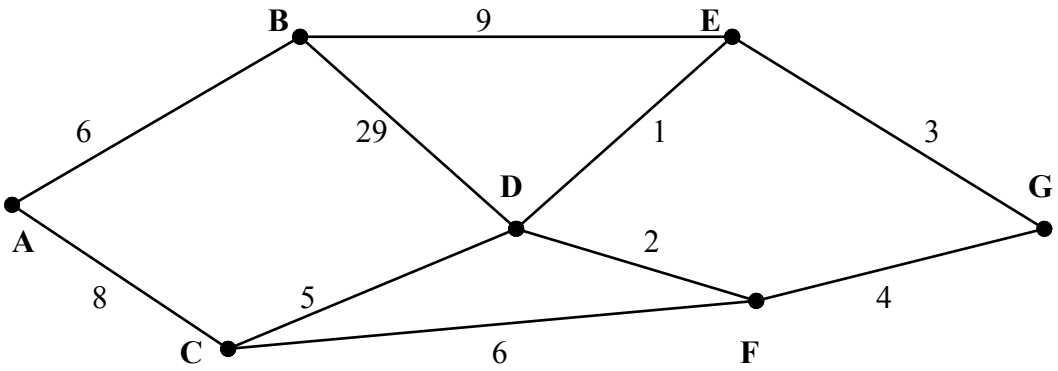
Each centre needs a link to the head office in Auckland but that link doesn't need to be a direct link to Auckland.



What is the minimum length of miracle thread that Brilliant needs to use in order to connect the centres?

ISP

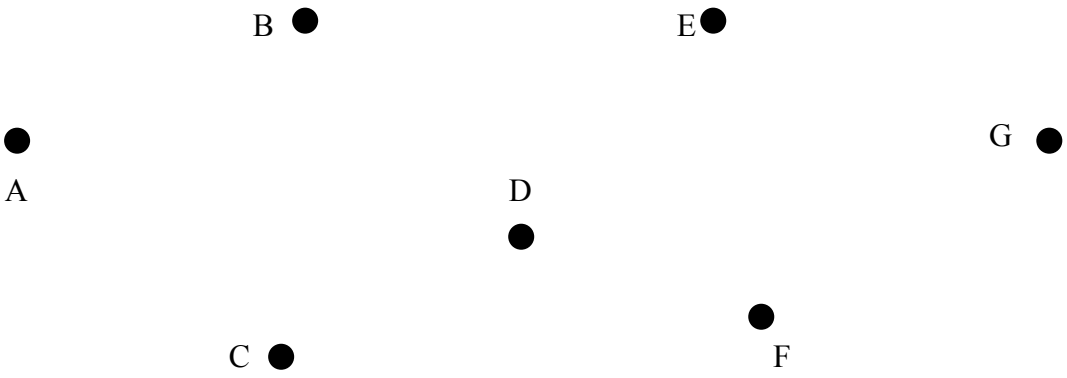
An Internet Service Provider (ISP) is intending to connect a series of towns to its new broad-band network. Each vertex represents a town and the numbers represent the costs (in millions of dollars) of constructing the links between the towns.



The company wants to establish the links at the smallest construction cost.

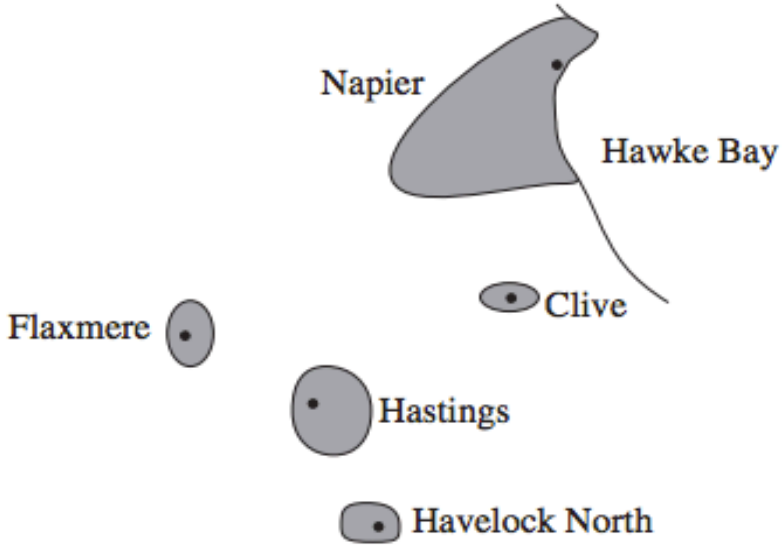
Draw a minimal spanning tree to help the ISP solve the problem.

What is the construction cost?



Drawing a Network

A business has offices in five towns in Hawkes Bay. They wish to connect the computers in each office with their own lines.



The table below shows the distances between the towns. For example, the distance between Flaxmere and Napier is 17.5 km.

Clive				
12.5	Flaxmere			
8.6	5.7	Hastings		
10.4	10.4	5.4	Havelock North	
9.6	17.5	16.4	19.6	Napier

a Draw a network to model this. Write in the distances on the arcs.

b Find and highlight the system of connections which uses the least amount of line to connect the computers in each town.

c The line will cost $10 000 per km to buy and lay.
What is the total cost of connecting the computers?

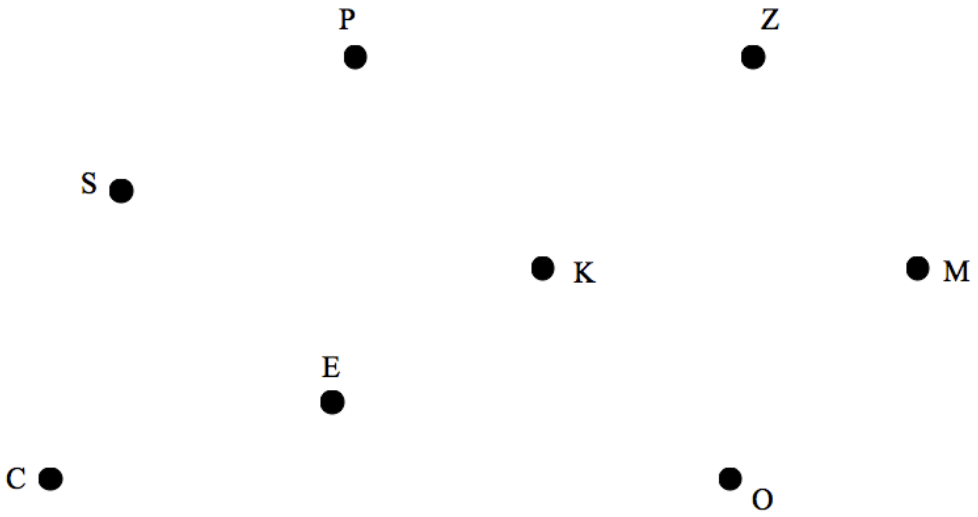
The ZOO

A new Zoo is being built and the developer is going to install a watering system.
Connections between the animals' enclosures are listed in the box below.

All lengths shown are in metres.

ENCLOSURE	Monkey (**M**)	Elephant (**E**)	Snake (**S**)	Zebra (**Z**)	Crocodile (**C**)	Penguin (**P**)	Ostrich (**O**)	Kiwi (**K**)
Monkey (**M**)				15			10	
Elephant (**E**)			21				35	12
Snake (**S**)					12	24		
Zebra (**Z**)						43	7	10
Crocodile (**C**)							73	
Penguin (**P**)								18
Ostrich (**O**)								20
Kiwi (**K**)								

Construct a network for the pipes connecting the animal enclosures, showing the lengths of the pipes on each arc.

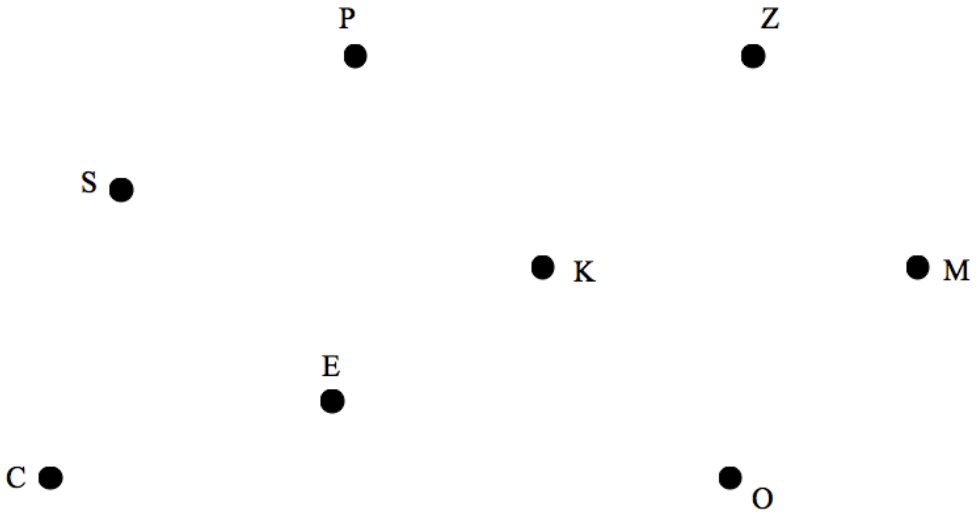


1 What is the shortest length of pipe needed to connect the snake enclosure (S) with the ostrich enclosure (O)?

2 List in order, the path of the shortest connection from the snake enclosure (S) to the ostrich enclosure (O)

3 The developer wishes to **minimise** the length of pipes used to connect all the enclosures.

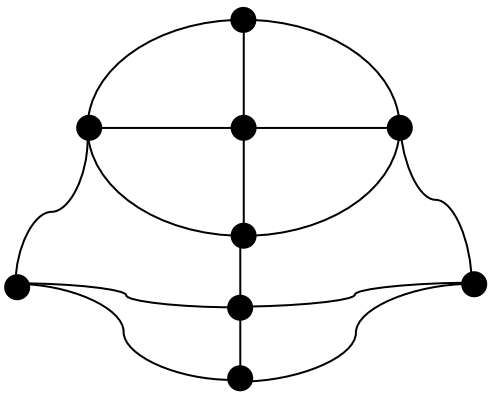
- Draw a **minimum spanning tree** for the network to show the length of pipe needed to connect all of the enclosures.
- Label all the connections clearly with their lengths.
- Find the minimum amount of pipe needed.



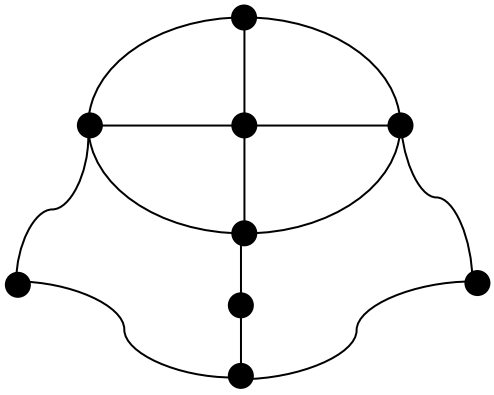
Minimum length of pipe needed = _____ metres.

5 Two potential path designs are being considered for the Zoo.
The path layout that will be chosen will be the design that is **traversible**.
Which path design will be chosen?

DESIGN A



DESIGN B



The Party Bus Company

The Party Bus Company offers tourist trips around Auckland City.
The buses have designated stops in the following locations. The distances between each stop are shown in the table

	Ferry Building	Kelly Tarltons	Mission Bay	Mt Eden	One Tree Hill	Museum	Zoo	Motat
Ferry Building		8				9		
Kelly Tarltons			3		14			
Mission Bay					12	4	10	11
Mt Eden						5		7
One Tree Hill							16	
Museum								
Zoo								2
Motat								

All distances shown are in kilometres.

Construct a network for the roads connecting the stops, showing the distances on each arc.

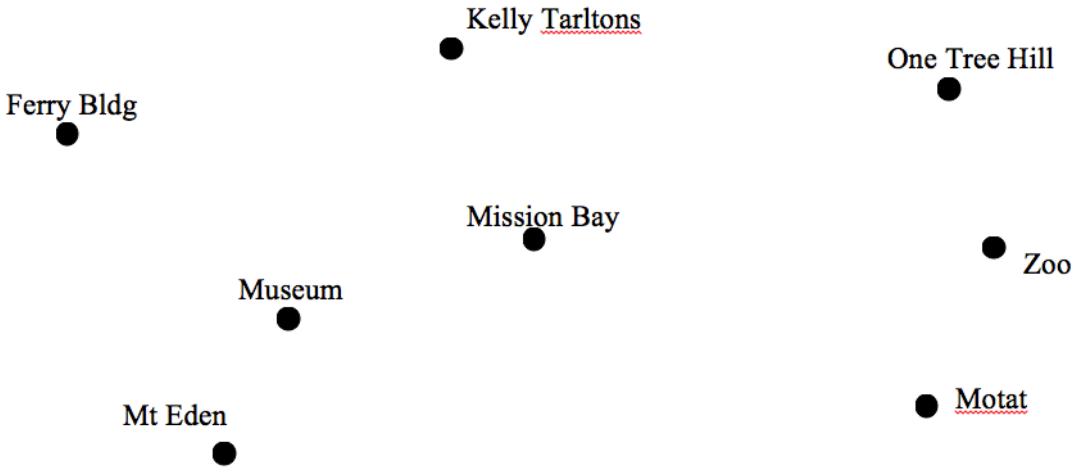
2. What is the shortest distance you could travel when travelling from the Ferry Building to Motat

_____ kilometres.

3 List in order, the shortest route from the Ferry Building to Motat

4 The Party Bus Company wishes to cut costs by minimising the distance the buses have to travel yet still stop at every location.

- Draw a minimum spanning tree for the network to show the distance needed to connect all of the locations.
- Label all the connections clearly with their lengths.
- State the minimum distance travelled.

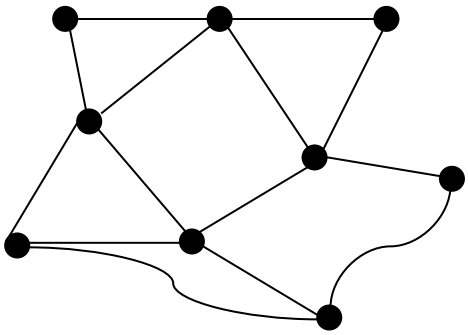


Minimum distance travelled = _____kilometres.

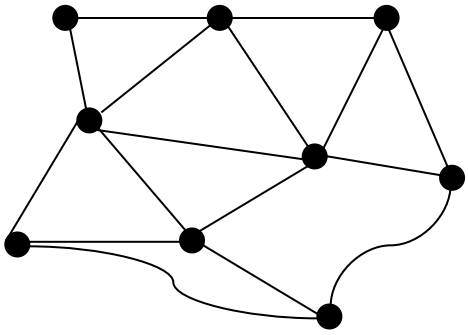
5 Two potential light rail networks are being considered by the Party Bus Company.
The network that will be chosen will be the design that is traversable.

- Which network will be chosen?
- Why is the light rail network design that you have chosen traversable?

DESIGN A

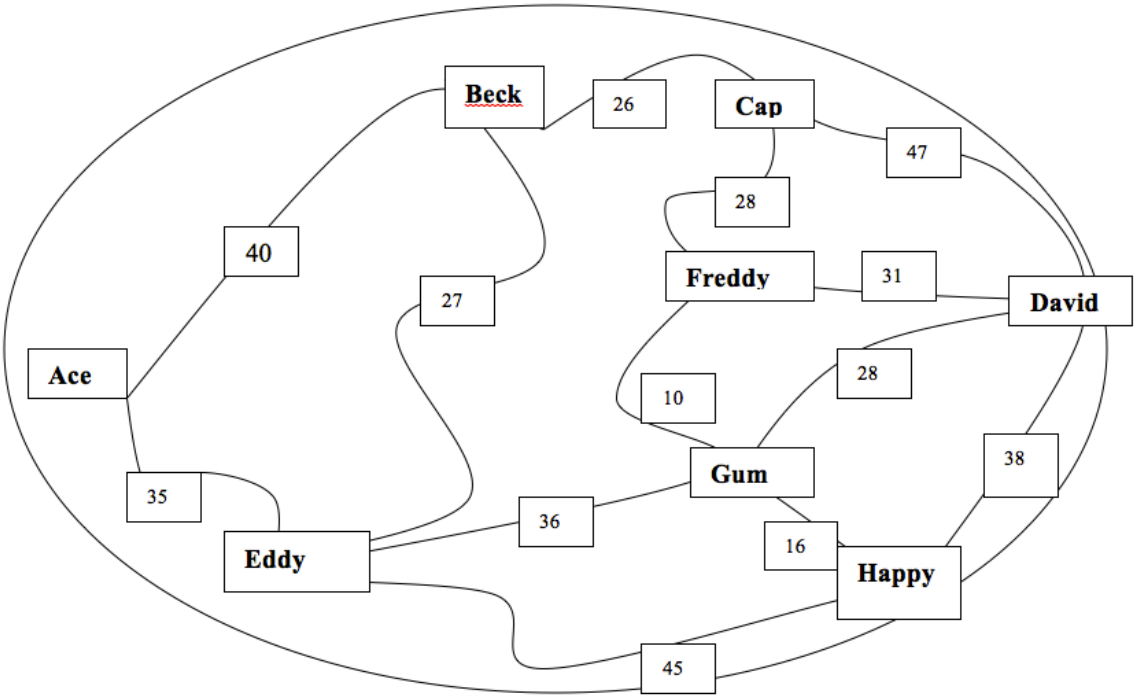


DESIGN B



Fiordland National Park

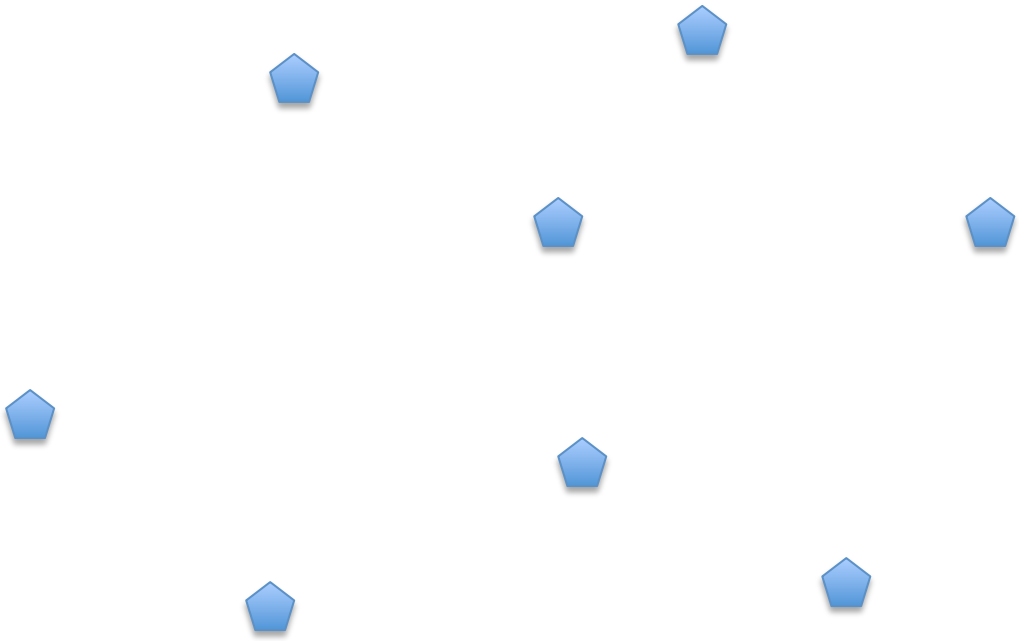
Fiordland National Park is in the South Island. There are lots of Department of Conservation huts that are linked by walking tracks as shown in the diagram. The outer ring shows the boundary of the park.



The distances between the huts in kilometres are shown on the map and in the following table

Ace							
40	Beck						
	26	Cap					
		47	David				
35	27			Eddy			
		28	31		Freddy		
			28	36	10	Gum	
			38			16	Happy

Construct a network for the tracks by joining the huts with lines
Label the huts with the first letter of its name eg B for Becks
Write the lengths of the tracks on the lines



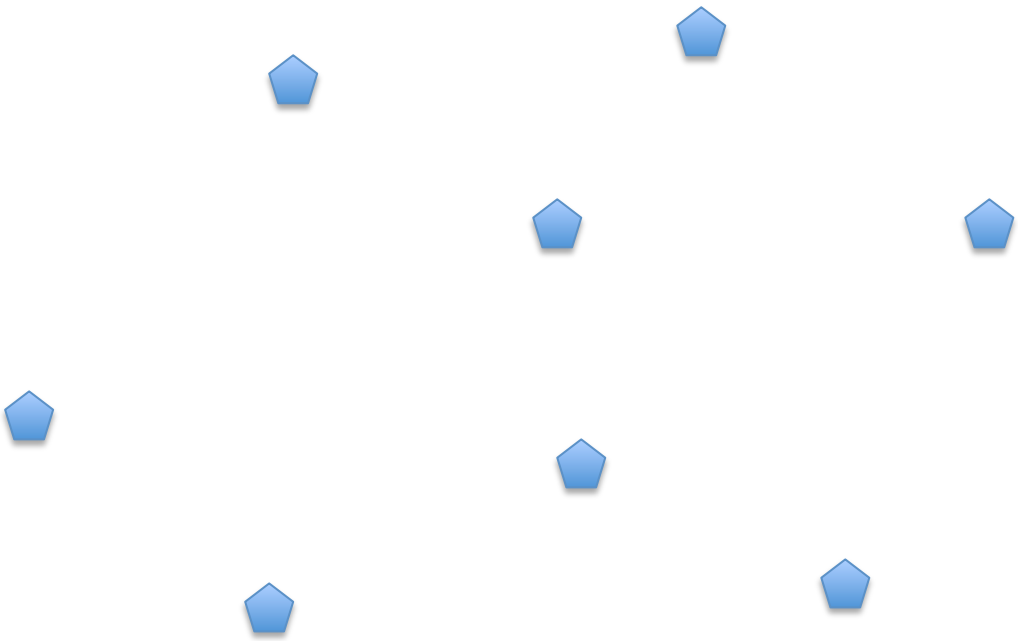
Find the shortest route from Becks to Happy huts

a) Write down the huts in order that you meet them to identify the shortest route from Becks to Happy

b) Add up each length on the line to find how far the shortest route is in Kilometres. .

Two trampers would like to walk all the tracks in the park without repeating any track.
They are happy to visit a hut more than once.

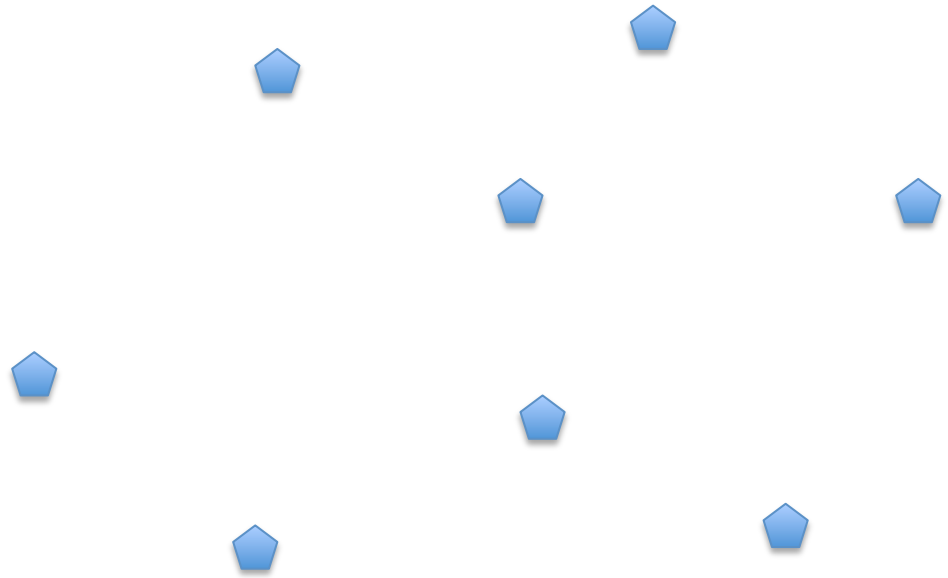
They would like to start at Ace hut and finish at Gum hut. Is this possible: explain your answer.



The Department of Conversation decided to close some tracks because the environment is very fragile, but they wish to leave a network of tracks so that it is possible to get to any of the huts

Draw the minimum spanning tree to show which tracks they should leave open

Label all the huts clearly and state the distances on the tracks.
State clearly the minimum length of track needed.



Spare grid