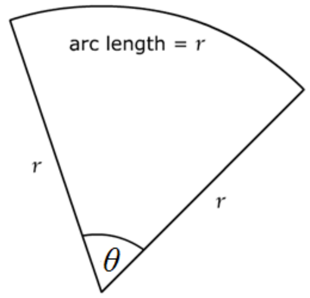
Year 13 Trigonometry Workbook

Radians and Degrees



θ = 1 radian

Angles can be measured in angles and radians.

A radian is the angle made by taking the radius and wrapping it along the edge of a circle.

If θ is in radians, then $\theta = \dfrac{\text{arc length}}{\text{radius}}$, (a number without a unit)

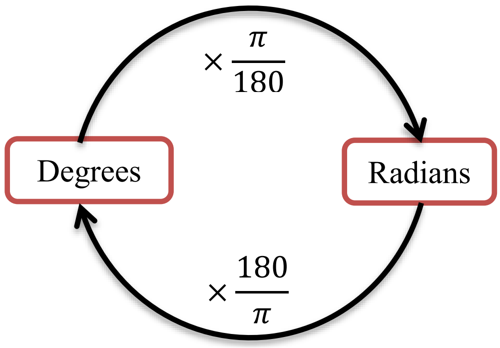
Converting between radians and degrees

The circumference of a whole circle is $2\pi r$. For a full circle:

In degrees $\quad \theta = 360^{o}$.

In radians $\quad \theta = \dfrac{\text{arc length}}{\text{radius}} = \dfrac{2\pi r}{r} = 2\pi$

So $\quad 360^{o} = 2\pi$ radians, or $180^{o} = \pi$ radians.



To convert from radians to degrees, multiply by $\dfrac{180}{\pi}$.

To convert from degrees to radians, multiply by $\dfrac{\pi}{180}$.

Example:
Convert 315^{o} to radians.
Leave your answer in terms of π.

Ans $\quad 315 \times \dfrac{\pi}{180} = \dfrac{315\pi}{180} = \dfrac{7\pi}{4}$

Example:
Convert $\dfrac{2\pi}{3}$ to degrees

Ans $\quad \dfrac{2\pi}{3} \times \dfrac{180}{\pi} = \dfrac{360\pi}{3\pi} = 120°$

Some useful conversions between degrees and radians are below, complete the table:

Angle in radians		$\dfrac{\pi}{4}$	$\dfrac{\pi}{3}$		π		
Angle in degrees	30^{o}			90^{o}		270^{o}	360^{o}

Exercise I: Angle Conversions

1. Convert the following angles from degrees to radians, leaving answers as multiples of π

   a. 90º____________________________________ b. 225º____________________________________

   c. 162º____________________________________ d. 15º____________________________________

2. Convert the following angles from radians to degrees, rounding to 2d.p. where necessary

   a. 2.3 rad____________________________________ b. $\frac{4\pi}{3}$ rad____________________________________

   c. $\frac{3\pi}{10}$rad____________________________________ d. 5.1 rad____________________________________

Graphs of Trigonometric Functions

Definitions
The **period** of a trig graph is the minimum cycle before a graph repeats itself
The **amplitude** of a trig graph is the maximum height either side of the central position
The **frequency** is the number of complete cycles the occur in 2π radians or 360 degrees (=$\frac{2\pi}{period}$)

The three main trig graphs are y = *sin* ***x***; y = *cos* ***x***; y = *tan* ***x***:

Properties of trig graphs

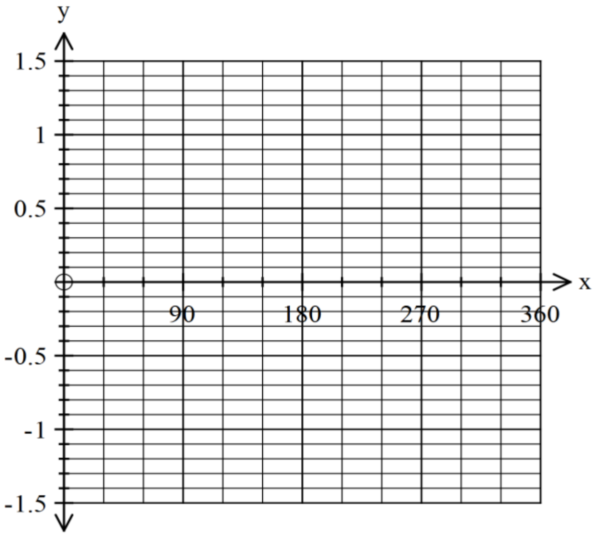
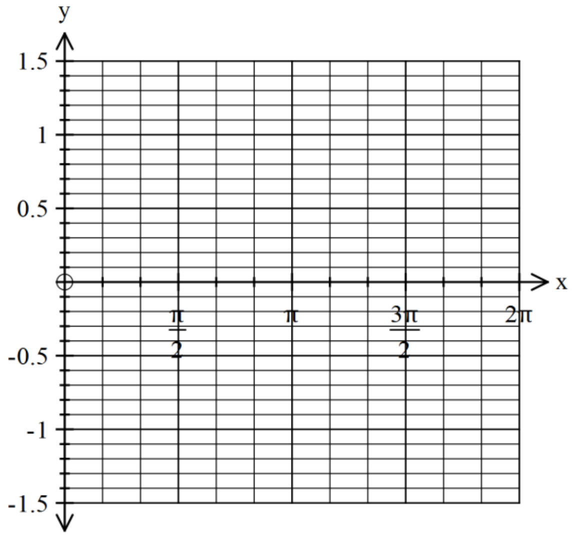
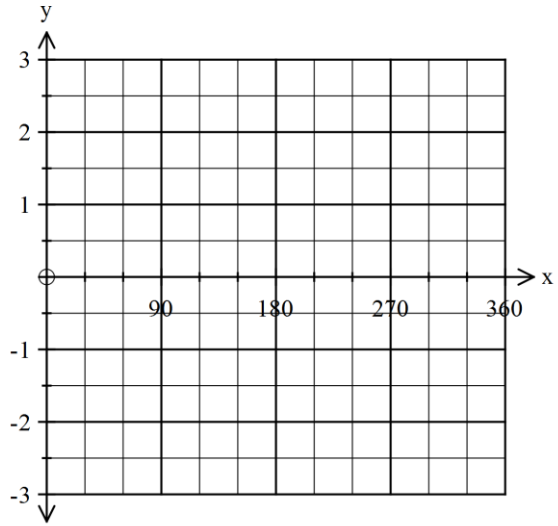
- y = *sin* ***x*** and y = *cos* ***x*** have a **period** of ***2π;*** y = *tan* ***x*** has a period of ***π***
- The **amplitude** of y = *sin* ***x*** and y = *cos* ***x*** is 1
- y = *tan* ***x*** is undefined for the values of *90°*, ***270°*** ($\frac{\pi}{2}, \frac{3\pi}{2}$ radians)- this is shown as **asymptotes** on graph
- y = *sin* ***x*** and y = *tan* ***x*** are **odd functions** (half turn rotational symmetry around the origin)
- y = *cos* ***x*** is an **even function** (y axis is a line of symmetry)
- For y = *sin* ***x*** and y = *cos* ***x*** the Domain is $x \in$ R ; the Range is -1< y < 1
- For y = *tan* ***x*** the Domain is $x \in$ R except for multiples of *90°* or $\frac{\pi}{2}$; the Range is $y \in$ R

Sketching trig graphs

- Graphs can be sketched in degrees or radians. It helps to use the GRAPH function on your graphics calculator.
- Your graphics calculator will automatically be in radians. To change the angle measure, press SHIFT, MENU. Scroll down to Angle and press F1 for DEG. Press EXIT to save.
- To see the entire graph, go SHIFT, F3 (**V-Window**). Change the following settings:

  X – min: 0　　　Y – min: −1.5
  　　max: 420　　　　max: 1.5

Exercise II: Table of Basic Trigonometric Graphs

Name of graph	***y* = sin *x***	***y* = cos *x***	***y* = tan *x***
Sketch			
***y* – intercept**	______________________	______________________	______________________
***x* – intercepts**	______________________	______________________	______________________
Amplitude	______________________	______________________	______________________
Period	______________________	______________________	______________________
Special features Odd/even/asymptotes	______________________ ______________________	______________________ ______________________	______________________ ______________________

Transformations of Trigonometric Graphs

The graphs of $y = \sin x$ and $y = \cos x$ can be transformed using

$$y = A \sin B(x+C)+D \quad \text{and}$$

$$y = A \cos B(x+C)+D$$

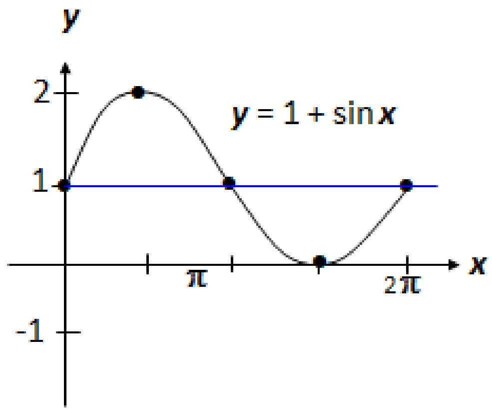
A ***changes the amplitude (vertical stretch)***

B ***changes the period (i.e.*** the number of times the graph occurs within a regular period***)***
so period = $\frac{2\pi}{B}$ or $\frac{360}{B}$

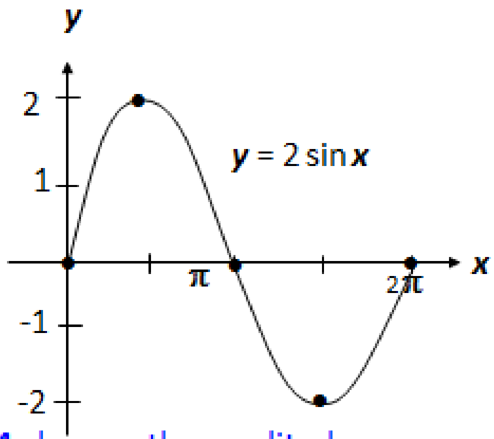
C ***translates the graph horizontally*** - moves the graph sideways

D ***translates the graph vertically*** – moves graph up and down – changes the location of the “midline”

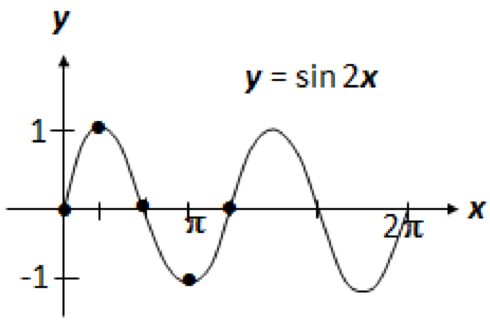
Summary:



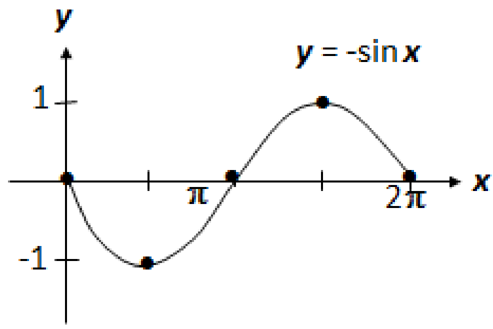
D translates function vertically
In this case $y = \sin x$ is moved up 1



A changes the amplitude
In this case the amplitude is 2



B changes the period
Period of $y = \sin x$ is 2π
Period of $y = \sin 2x$ is $2\pi \div 2 = \pi$

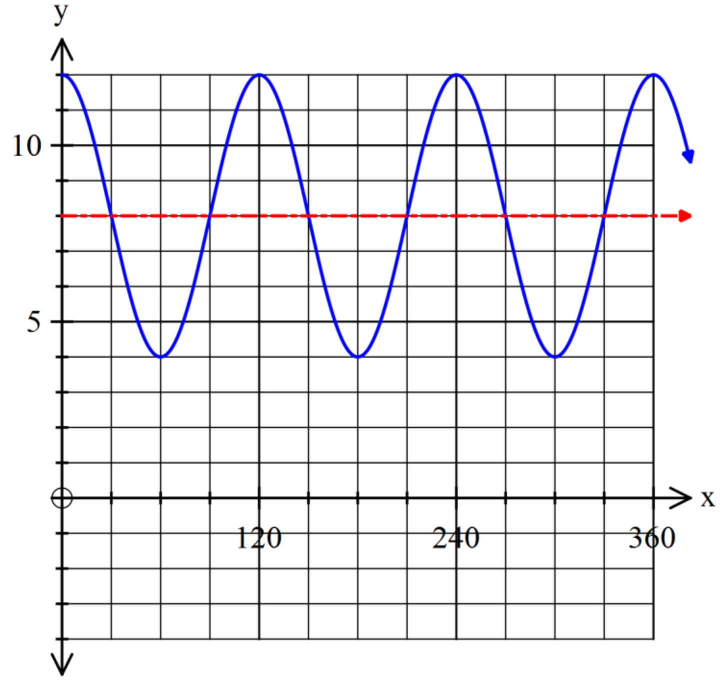


negative sign in front of the function reflects the graph in the x-axis

Examples

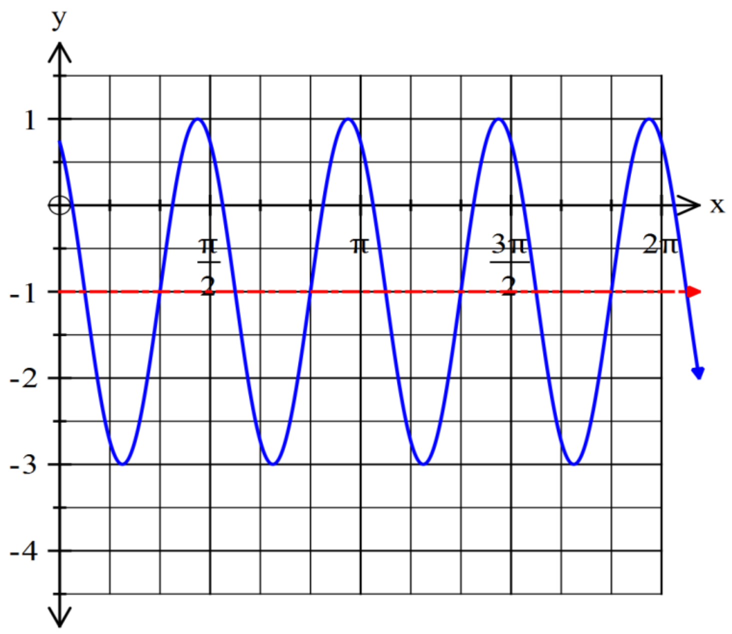
A Sketch the graph $y = 4\cos(3x)+8$ (in degrees) identifying key features

A = 4 So amplitude = 4
B = 3 So each period = $\frac{360}{3} = 120°$
C – there is no horizontal shift
D = 8 Graph moves up 8 ("midline is y = 8)
Maximum point = 8 + 4 = 12 Minimum = 8 – 4 = 4



B Identify the key features of $y = 2\sin 4(x - \frac{\pi}{3}) - 1$ given that the equation is in radians.

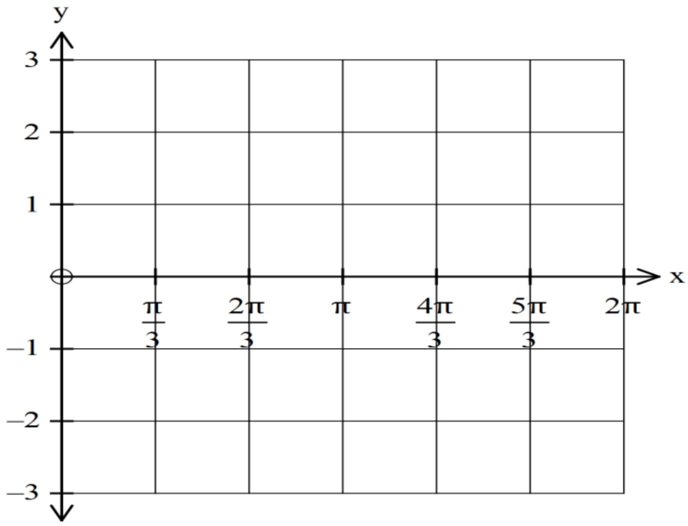
A = 2 So amplitude = 2
B = 4 So each period = $\frac{2\pi}{4} = \frac{\pi}{2}$
C – graph has moved $\frac{\pi}{3}$ to the right compared to $y = \sin x$
D =-1 Graph moves down 1 ("midline is y = -1)
Maximum point = -1 + 2 = 1 Minimum = -1 – 2 = -3



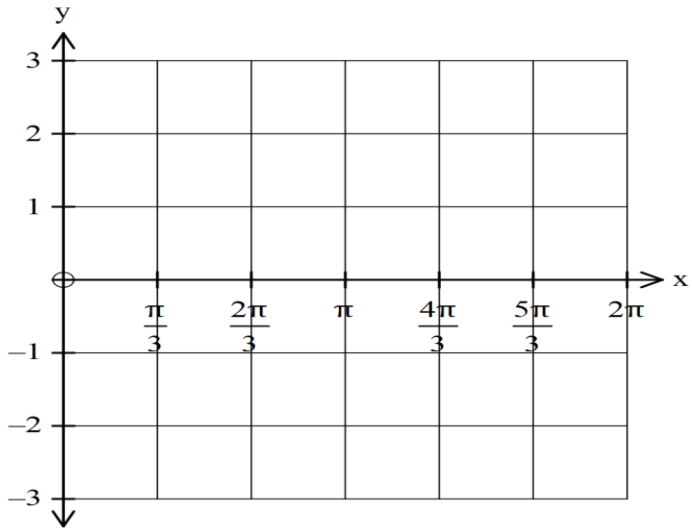
Exercise III: Finding Key Points and Sketching Transformed Graphs

Find the amplitude, period and any horizontal or vertical shift then and then sketch on the grid.

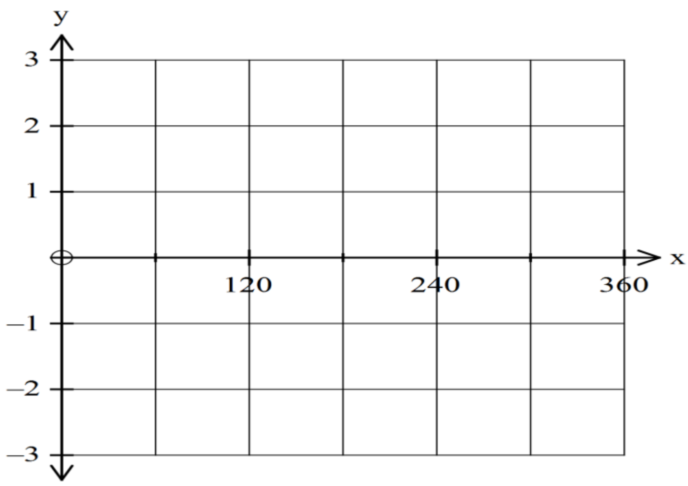
1. $y = 2\cos 3x$



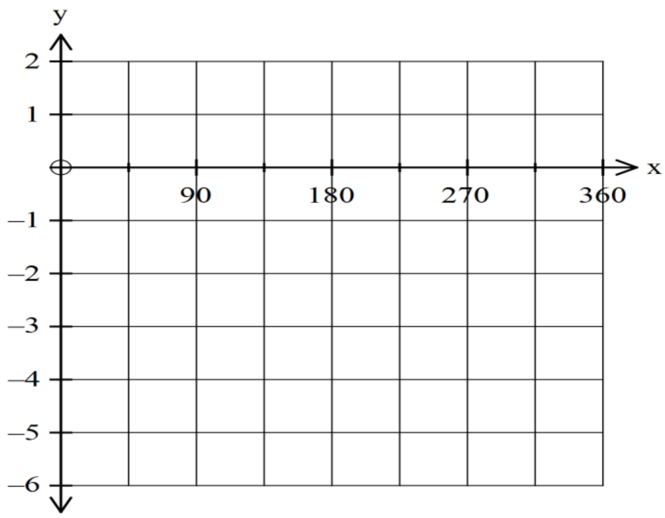
2. $y = \cos 2x + 1$



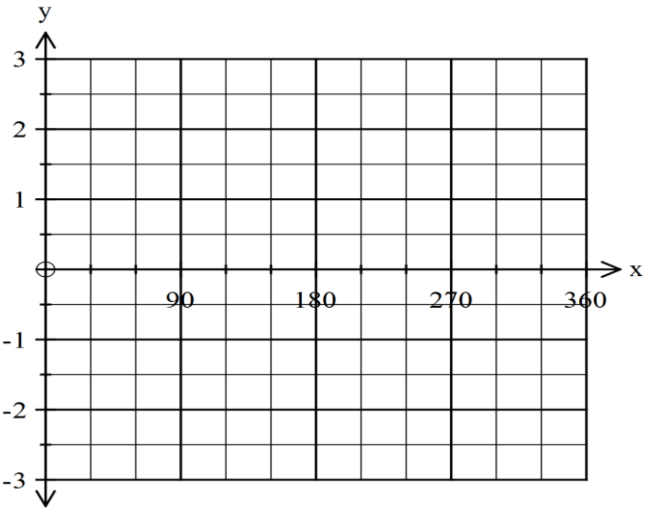
3. $y = \frac{1}{2}\cos 3x - 2$



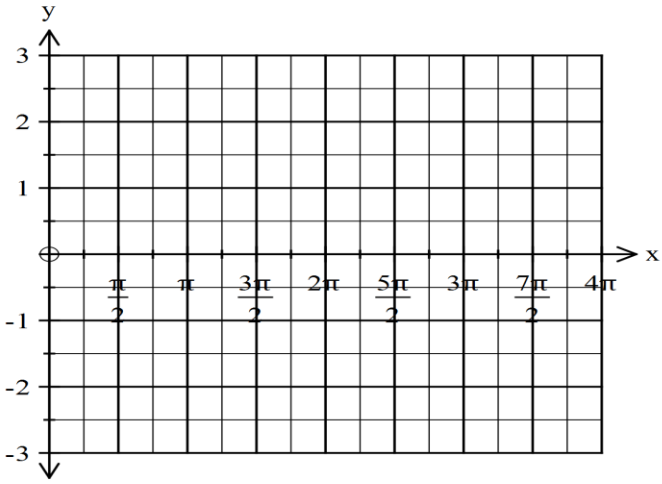
4. $y = 4\cos 2(x - 30) - 2$



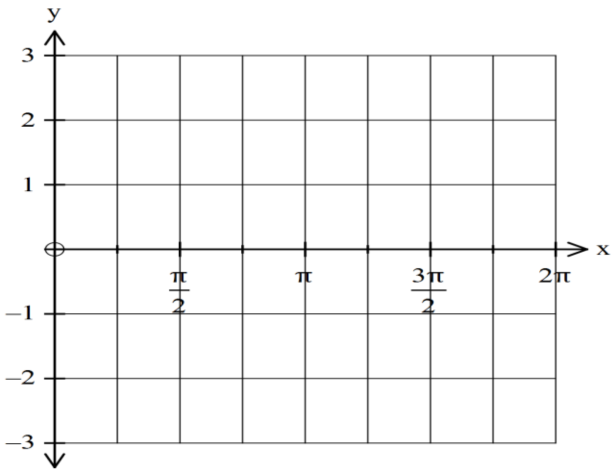
5. $y = 2\cos(x + 45) + 1$



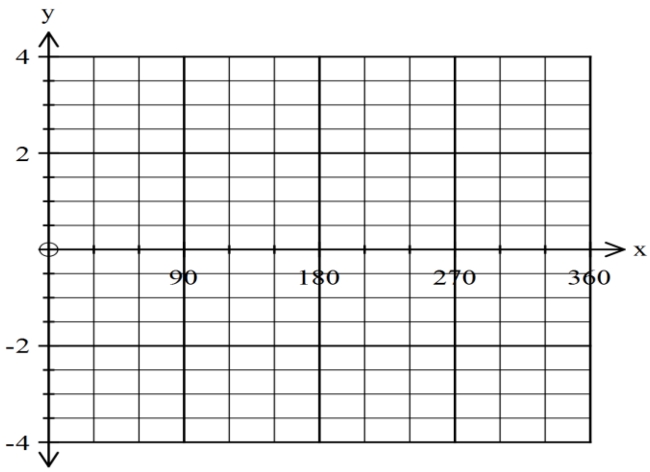
6. $y = \sin\frac{1}{2}x - 1$



7. $y = 3\sin 2x$



8. $y = 3\sin 3(x + 20) + 1$



Writing Equations from Trigonometric Graphs

The general format of the curve will either be:

$$y = A \sin B(x + C) + D \quad \text{or} \quad y = A \cos B(x + C) + D$$

Note: A cosine graph is a shifted sine graph (and vice versa), so it does not matter which equation you choose. The only difference will be your value of *C*.

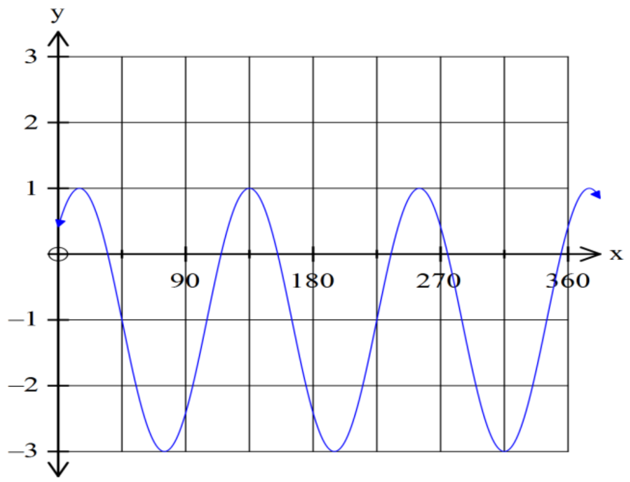
Find D: Draw a horizontal line halfway between the maximum and minimum value and calculate the distance from the *x*–axis.

Find A: Measure the amplitude (half the distance between max and min values). Add a negative sign if the graph is inverted.

Find B: Measure the period along the horizontal line. B = = $\frac{2\pi}{\text{period}}$ or $\frac{360}{\text{period}}$

Find C: Find the horizontal shift; either by inspection or by substituting a known value into equation

Example: Write the equation of this trigonometric graph



1. *D:* max value = 1; min value = −3
   D = halfway between 1 and −3 = −1

2. $A = \frac{\text{max value} - \text{min value}}{2} = \frac{1 - -3}{2} = 2$

3. *B*: period = 120
   $B = \frac{360}{\text{period}} = \frac{360}{120} = 3$

4. Decide whether to use sine or cosine. In this case, cosine is chosen.

   $y = A \cos B(x + C) + D$
   $y = 2 \cos 3(x + C) - 1$

5. *C:* difficult to find C by inspection so substitute a co-ordinate into the equation

   Use (*x*, *y*) co-ordinate (45, −1)

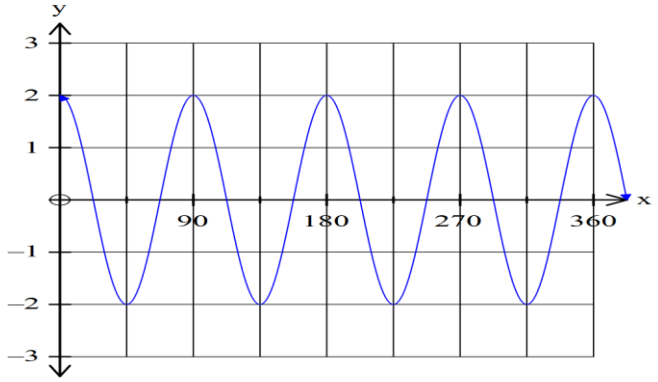
   $y = 2 \cos 3(x - C) - 1$
   $-1 = 2 \cos 3(45 - C) - 1$
   $0 = 2 \cos 3\,(45 - C)$
   $0 = \cos 3\,(45 - C)$
   $90 = 3\,(45 - C)$
   $30 = 45 - C$
   $15 = C$

The equation of this graph is $y = 2 \cos 3\,(x - 15) - 1$

Exercise IV: Writing Equations from Trigonometric Graphs

Write trigonometric equations for the following graphs. Check your solution using your graphics calculator.

1



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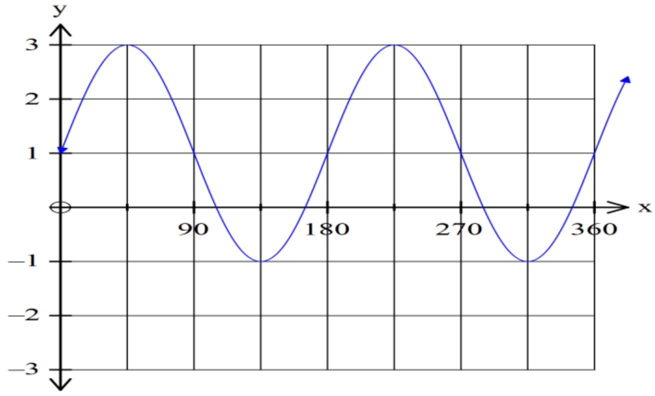

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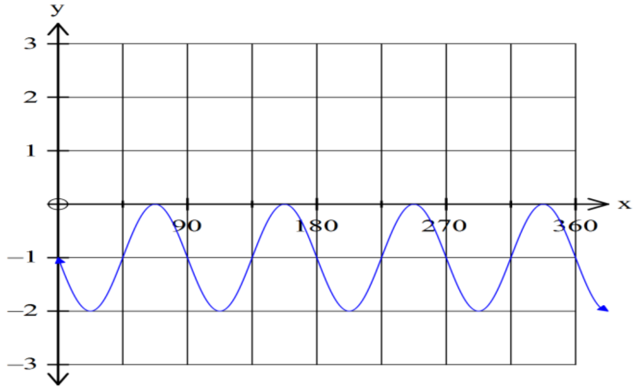

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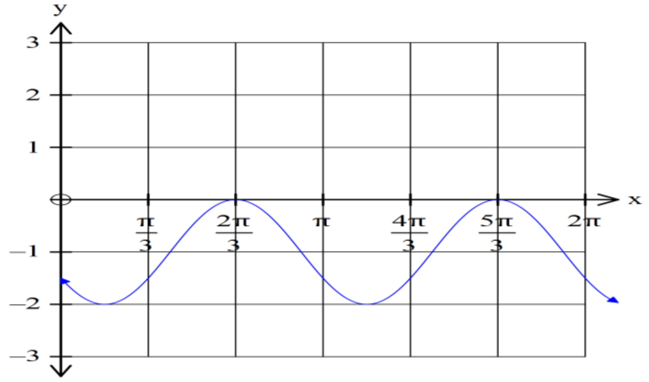

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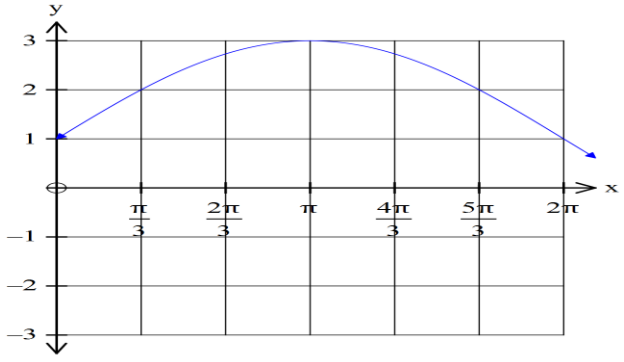

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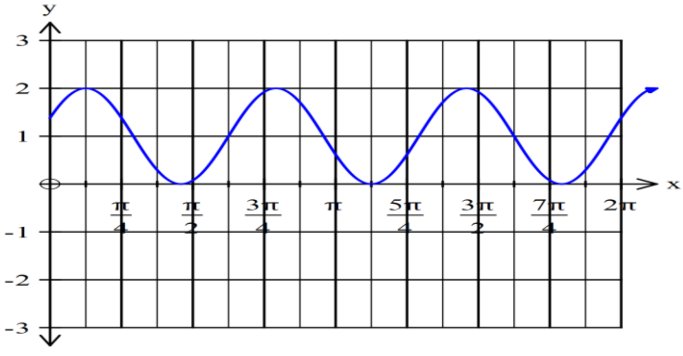


Solving Trigonometric Equations

A trigonometric equation is where we want to find where a trigonometric function intersects with a horizontal line.

Example: Solve cos $x = 0.5$, $0 \leq x \leq 360^{\circ}$

Graphics Calculator

Check that the angle measure is in degrees.

The **V-window** should be set to
X – min: 0 X – max: 360°
Y – min: −1 Y – max: 1

In the graph function, draw:
- $y = \cos x$
- $y = 0.5$

Find the intercepts by pressing SHIFT, F5, F5 (**ISCT**)

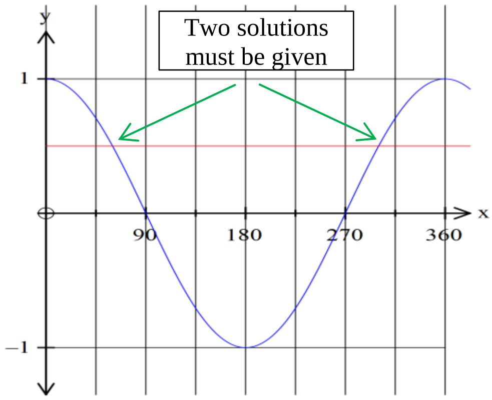
Algebraically

$\cos x = 0.5$
$x = \cos^{-1} 0.5$
$x = 60^{\circ}$

Since the cosine graph is symmetrical between 0 and 360°, another solution to cos x = 0.5 exists.

$x = 360 - 60 = 300^{\circ}$

Therefore, $x = 60^{\circ}$ and 300°



Always draw a diagram when solving trigonometric equations

When solving trigonometric equations algebraically, the diagram must be drawn in the step directly before the operation $\sin^{-1}$ or $\cos^{-1}$ is used.

All solutions in the specified domain must be given.

Example: Solve 2 cos $x = 0.5$, $0 \leq x \leq 360^{\circ}$

Graphics Calculator

The **V-window** should be set to
X – min: 0 X – max: 360°
Y – min: −2 Y – max: 2

The amplitude has altered here

In the graph function, draw:
- $y = 2\cos x$
- $y = 0.5$

Find the intercepts by pressing SHIFT, F5, F5 (**ISCT**)

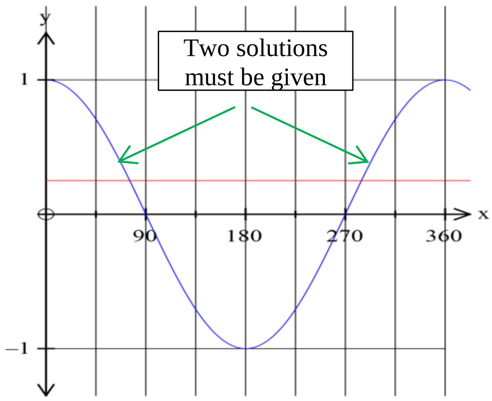
Algebraically

$2\cos x = 0.5$
$\cos x = 0.25$ → Draw diagram
$x = \cos^{-1} 0.25$
$x = 75.52^{\circ}$

Since the cosine graph is symmetrical between 0 and 360°, another solution to 2cos x = 0.5 exists.

$x = 360 - 75.52 = 284.48^{\circ}$

Therefore, $x = 75.52^{\circ}$ and 284.48°



Always draw a diagram when solving trigonometric equations

Example: Solve cos 2x = 0.5, $0 \leq x \leq 360^o$

Graphics Calculator

The **V-window** should be set to
X – min: 0 X – max: 360°
Y – min: −1 Y – max: 1

The period has altered to 180°

In the graph function, draw:
- $y = \cos 2x$
- $y = 0.5$

Find the intercepts by pressing SHIFT, F5, F5 (**ISCT**)

Algebraically

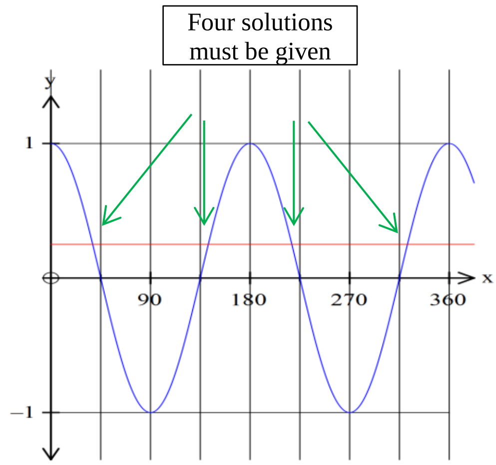
$\cos 2x = 0.5$
$2x = \cos^{-1} 0.5$
$2x = 60^o$
$x = 30^o$

Draw diagram →

Since the cosine graph is symmetrical between 0 and 180° and 180° and 360°, three other solutions must exist.

$x = 180 – 60 = 120^o$
$x = 180 + 60 = 240^o$
$x = 360 – 60 = 300^o$

Therefore, $x = 60^o, 120^o, 240^o$ and 300^o



Always draw a diagram when solving trigonometric equations

Example: Solve cos (x + 20) = 0.5, $0 \leq x \leq 360^o$

Graphics Calculator

The **V-window** should be set to
X – min: 0 X – max: 360°
Y – min: −1 Y – max: 1

There is a horizontal shift in the graph

In the graph function, draw:
- $y = \cos(x + 20)$
- $y = 0.5$

Find the intercepts by pressing SHIFT, F5, F5 (**ISCT**)

Algebraically

Let $w = x + 20$

$\cos w = 0.5$
$w = \cos^{-1} 0.5$
$w = 60^o$

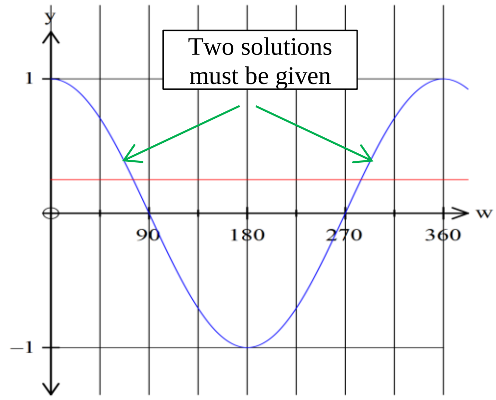
Draw diagram →

Since the cosine graph is symmetrical between 0 and 360°, another solution to cos w = 0.5 exists.

$w = 360 – 60 = 300^o$

To calculate x, substitute $x + 20 = w$ back in.

$x + 20 = 60^o$ $x = 40^o$
$x + 20 = 300^o$ $x = 280^o$



Always draw a diagram when solving trigonometric equations

Example: Solve $\cos x + 2 = 1.5,\ 0 \le x \le 360^\circ$

Graphics Calculator

The **V-window** should be set to
X – min: 0 X – max: 360°
Y – min: 1 Y – max: 3

There is a vertical shift in the graph

In the graph function, draw:
- $y = \cos x + 2$
- $y = 0.5$

Find the intercepts by pressing SHIFT, F5, F5 (**ISCT**)

Algebraically

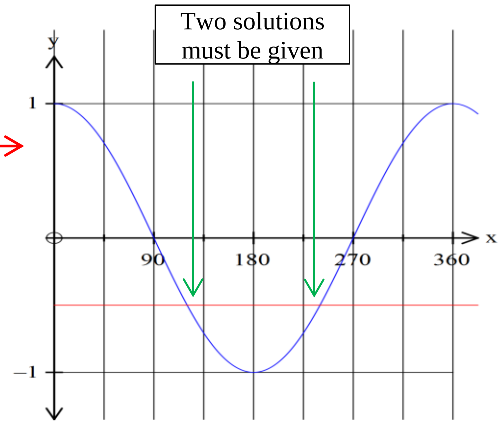
$\cos x + 2 = 1.5$
$\cos x = -0.5$
$x = \cos^{-1}(-0.5)$
$x = 120^\circ$

Draw diagram

Since the cosine graph is symmetrical between 0 and 360°, another solution must exist

$x = 360 - 120 = 240^\circ$

Therefore, $x = 120^\circ$ and 240°



Always draw a diagram when solving trigonometric equations

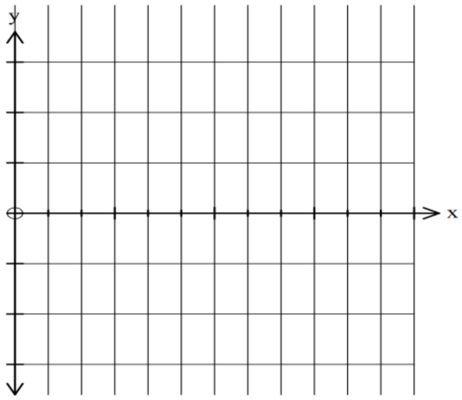
Exercise V: Solving Trigonometric Equations

Using algebraic methods solve the following trigonometric equations in the specified domain. Space has been provided for you to sketch a diagram of the trigonometric function and the line it intersects with.

Check your solutions using your graphics calculator.

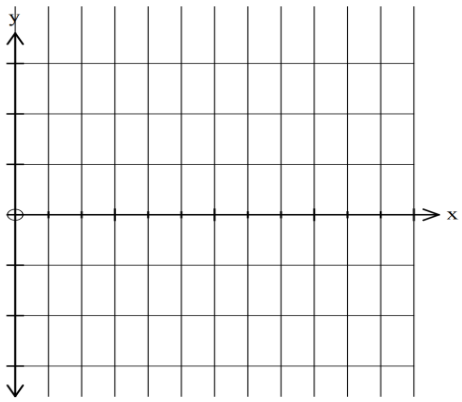
ONE

$\cos x = 0.3,\ 0 \le x \le 2\pi$



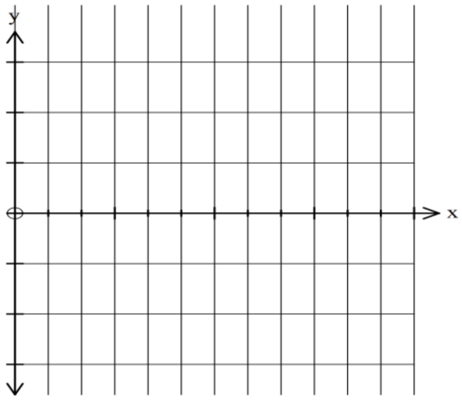
TWO

$\cos 2x = 0.3,\ 0 \le x \le 2\pi$



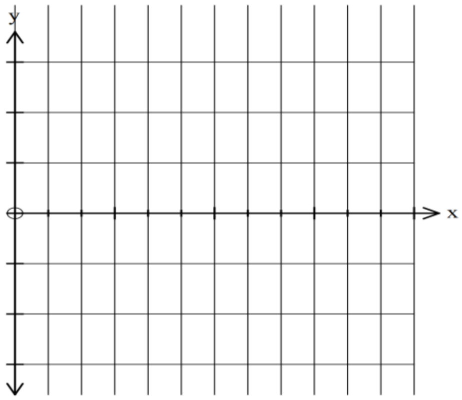
THREE

$\cos(x + 1.2) = 0.3,\ 0 \le x \le 2\pi$



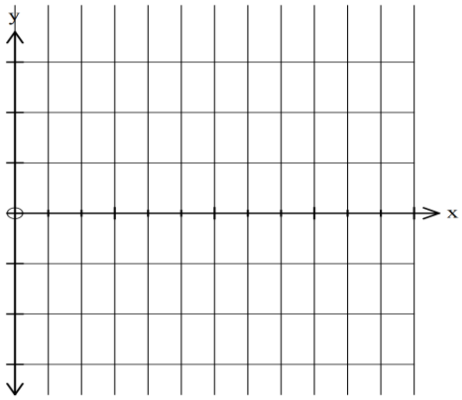
FOUR

$\cos x - 1 = -1.2,\ 0 \le x \le 2\pi$



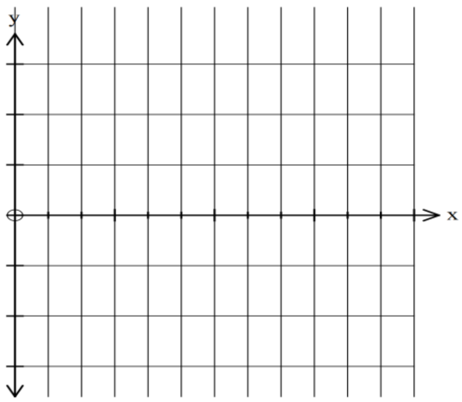
FIVE

$\sin x = -0.45,\ 0 \le x \le 360^{o}$



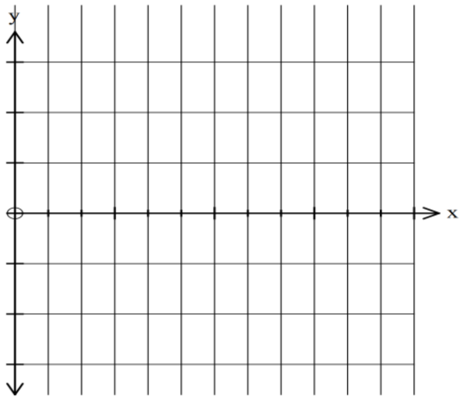
SIX

$\sin 3x = 0.2,\ 0 \le x \le 180^{o}$



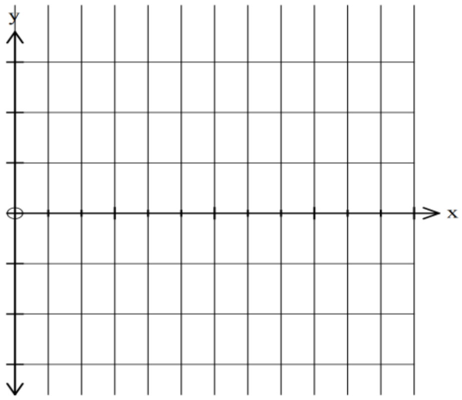
SEVEN

$\sin(x - 2.1) = 0.62,\ 0 \le x \le 2\pi$



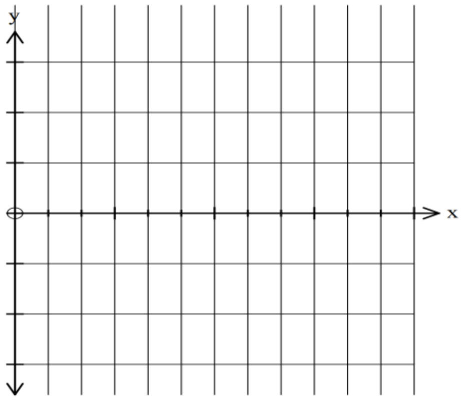
EIGHT

$\sin x + 2 = 1.86,\ 0 \le x \le 2\pi$



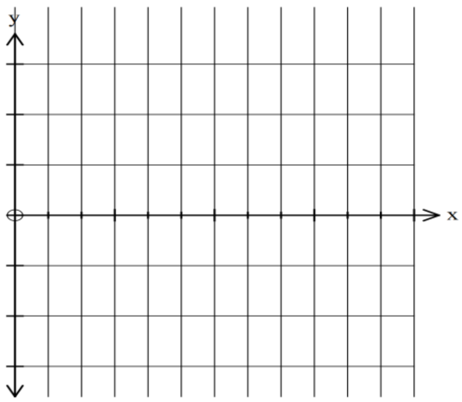
NINE

$2 \sin x = -1.8,\ 0 \le x \le 360^{o}$



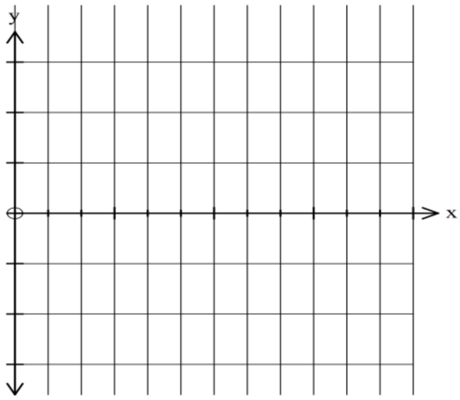
TEN

$\cos 4x = 0.3,\ 180^{o} \le x \le 360^{o}$



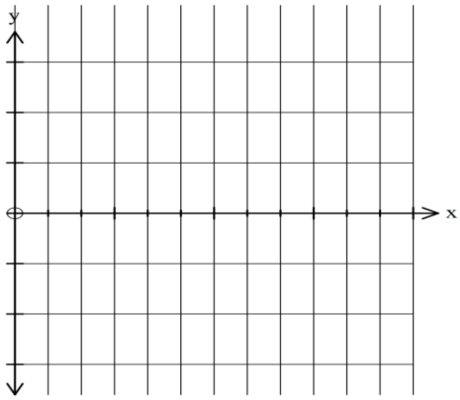
$3\cos x - 2 = -0.5, \ -180^{\circ} \leq x \leq 180^{\circ}$

ELEVELN



$2\sin\left(x - \frac{\pi}{3}\right) = 1.18, \ 0 \leq x \leq 2\pi$

TWELVE



Solving Trigonometric Equations 2

To solve trigonometric equations where there are multiple transformations to the trigonometric function, we want to remove as many transformations as possible, before sketching the diagram and solving.

Example: Solve $3\cos 2(x + 20) + 2 = 4.2, \ 0 \leq x \leq 180^{\circ}$

Graphics Calculator

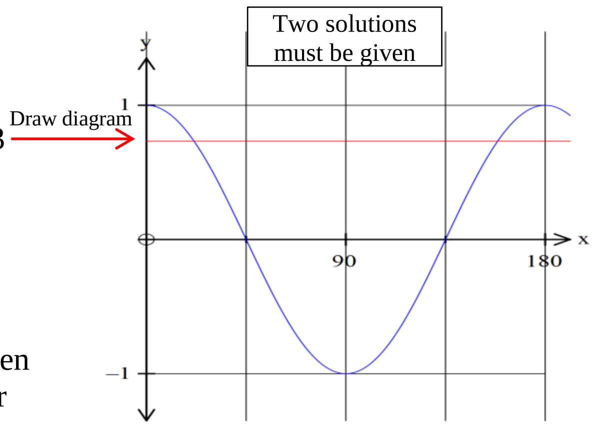
The **V-window** should be set to
X – min: 0 X – max: 180°
Y – min: 2 – 3 = –1
Y – max: 2 + 3 = 5

In the graph function, draw:
- $y = 3\cos 2(x + 30) + 2$
- $y = 4.2$

Find the intercepts by pressing SHIFT, F5, F5 (**ISCT**)

Algebraically

$3\cos 2(x + 20) + 2 = 4.2$
$3\cos 2(x + 20) = 2.2$
$\cos 2(x + 20) = 0.7333$
Let $w = x + 20$
$\cos 2w = 0.7333$
$2w = 42.83^{\circ}$
$w = 21.42^{\circ}$



Due to the symmetry between 0° and 180°, there is another solution.

$w = 180 - 21.42 = 158.58^{\circ}$

$x = 21.42 - 20 = 1.42^{\circ}$ and $158.58 - 20 = 138.58^{\circ}$

Always draw a diagram when solving trigonometric equations

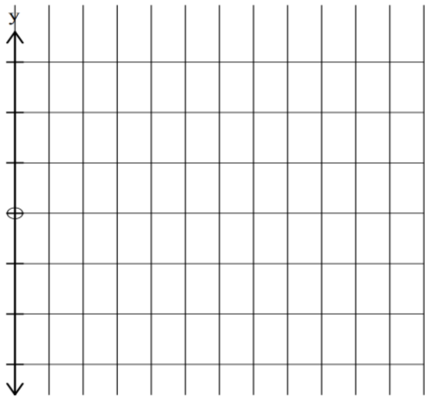
Exercise VI: Solving Trigonometric Equations 2

Using algebraic methods solve the following trigonometric equations in the specified domain. Space has been provided for you to sketch a diagram of the trigonometric function and the line it intersects with.

Check your solutions using your graphics calculator.

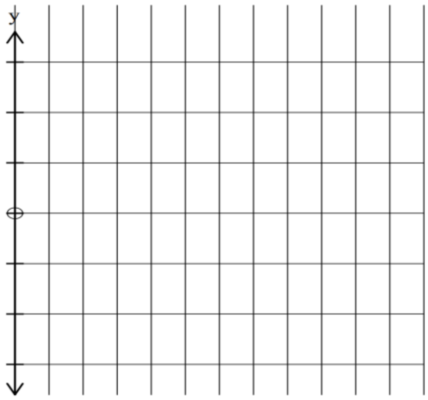
ONE

$5 \sin (x - 15) + 4 = 1.8,\ 0 \le x \le 360^{o}$



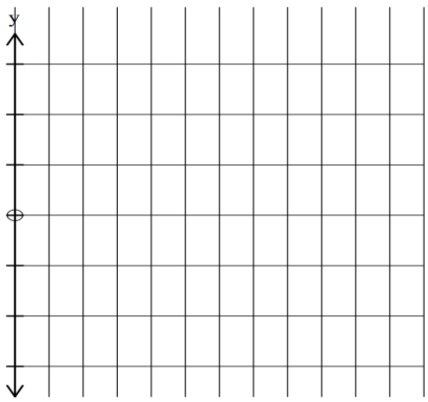
TWO

$3\sin 2x - 3 = -1.2,\ 0 \le x \le 2\pi$



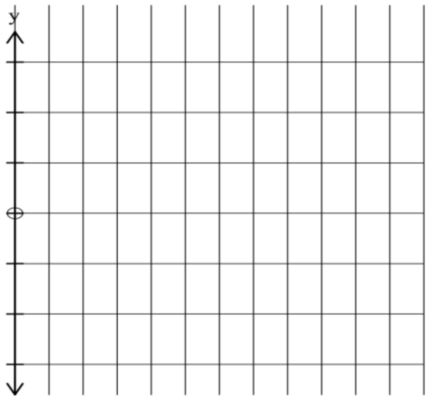
THREE

$7 - 2\cos 3\left(x + \frac{\pi}{6}\right) = 6,\ 0 \le x \le \pi$



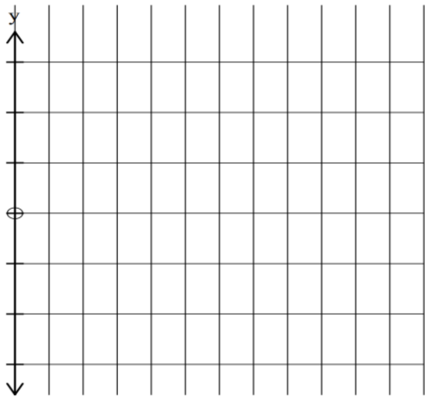
FOUR

$12\cos 4\left(x+\frac{\pi}{8}\right)-9=2.8,\ 0\le x\le \pi$



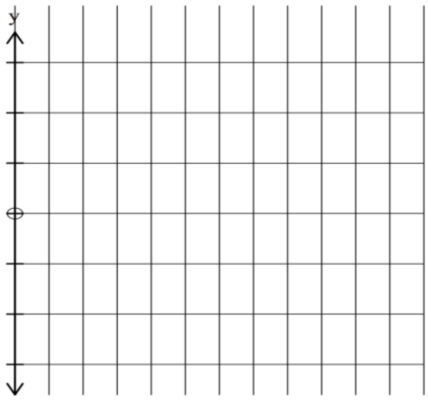
FIVE

$15 - \sin 2(x - 45) = 14.2,\ 0 \le x \le 180^{o}$



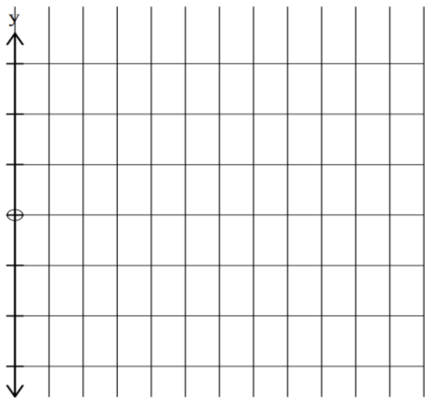
SIX

$14\sin\frac{1}{2}(x+15)+3=10,\ 0\le x\le 720^{o}$



SEVEN

$5\cos 3\,(x - 1.25) + 20 = 24,\ 0 \le x \le 2\pi$



General Solutions

In the previous section, we solved trigonometric equations in a specific domain by rearranging and using the symmetry found in the graphs.

However, if we wish to find all the solutions to a trigonometric equation, the number of solutions is infinite.

We can use general solutions to show all solutions for a trigonometric equation. These can also be found on your formula sheet.

If $\sin\theta = \sin\alpha$ then $\theta = n\pi + (-1)^n\alpha$
If $\cos\theta = \cos\alpha$ then $\theta = 2n\pi \pm \alpha$
If $\tan\theta = \tan\alpha$ then $\theta = n\pi + \alpha$
where n is any integer

If $\sin\theta = \sin\alpha$ then $\theta = 180n + (-1)^n\alpha$
If $\cos\theta = \cos\alpha$ then $\theta = 360n \pm \alpha$
If $\tan\theta = \tan\alpha$ then $\theta = 180n + \alpha$
where n is any integer

The general solutions are formed as a result of the symmetry and periodicity found in each graph.

Example: Give the general solution of $2\sin 3\left(x + \frac{\pi}{2}\right) + 5 = 4$

Step 1
Remove any vertical shift and amplitude change from the trigonometric function

$$2\sin 3\left(x + \frac{\pi}{2}\right) + 5 = 4$$
$$2\sin 3\left(x + \frac{\pi}{2}\right) = -1$$
$$\sin 3\left(x + \frac{\pi}{2}\right) = -\frac{1}{2}$$

Step 2
Set $\alpha = \sin^{-1}$ ….

The value of α is always the value that has had an inverse trigonometric function applied to it.

$$3\left(x + \frac{\pi}{2}\right) = \sin^{-1}\left(-\frac{1}{2}\right)$$
$$3\left(x + \frac{\pi}{2}\right) = -0.5236$$

This means that $\alpha = -0.5236$

Step 3
Substitute into the general equation.
In this case $\theta = n\pi + (-1)^n\alpha$ must be used as the function is sine. θ is equal to $3\left(x + \frac{\pi}{2}\right)$

$$3\left(x + \frac{\pi}{2}\right) = n\pi + (-1)^n(-0.5236)$$

Step 4
Rearrange general equation to obtain $x = \ldots$
Notice that the $-\frac{\pi}{2}$ is not simplified, as (-0.1745) switches between being a positive and a negative depending on the value of n.

$$x + \frac{\pi}{2} = \frac{n\pi}{3} + (-1)^n(-0.1745)$$
$$x = \frac{n\pi}{3} + (-1)^n(-0.1745) - \frac{\pi}{2}$$

Step 5
Substitute values of n into the general equation to find particular numerical solutions. General solutions can be used to find particular solutions in a specified domain.
Check solutions using a graphics calculator (as shown in previous section)

If $n = 0$, then $x = \frac{(0)\pi}{3} + (-1)^{(0)}(-0.1745) - \frac{\pi}{2} = -1.745$
If $n = 1$, then $x = \frac{(1)\pi}{3} + (-1)^{(1)}(-0.1745) - \frac{\pi}{2} = -0.349$
If $n = 2$, then $x = \frac{(2)\pi}{3} + (-1)^{(2)}(-0.1745) - \frac{\pi}{2} = 0.349$
If $n = 3$, then $x = \frac{(3)\pi}{3} + (-1)^{(3)}(-0.1745) - \frac{\pi}{2} = 1.745$
If $n = 4$, then $x = \frac{(4)\pi}{3} + (-1)^{(4)}(-0.1745) - \frac{\pi}{2} = 2.443$
If $n = 5$, then $x = \frac{(5)\pi}{3} + (-1)^{(5)}(-0.1745) - \frac{\pi}{2} = 3.840$

Exercise VII: General Solutions

Write the general solution of the following equations and give the *x*-values for $n = 0, 1, 2$ and 3. Check your solutions on your graphics calculator.

$8 \cos 3x = 6$ in radians

ONE

$2 \sin x - 2 = -0.5$ in degrees

TWO

$\sin 3(x + 180^{o}) + 15 = 15.5$ in degrees

THREE

$$9\cos\left(x - \frac{4\pi}{5}\right) - 2 = 6.4 \text{ in radians}$$

FOUR

$$17\cos 2(x - 62^{o}) + 25 = 10.5 \text{ in degrees}$$

FIVE

$$45 - 6\sin\frac{\pi}{20}(x + 7.5) = 41.6 \text{ in radians}$$

SIX

Applications of Trigonometric Graphs

In this section, we use all the skills taught on writing equations of trigonometric graphs and solving trigonometric equations to answer problems in contextual situations.

e.g. A team of biologists have discovered a new creature in the rain forest. They note the temperature of the animal appears to vary sinusoidally over time.

A maximum temperature of 50°C occurs 15 minutes after they start their examination. A minimum temperature of 35°C occurs 28 minutes later.

a) Using the information given, form an equation to model the temperature of the animal over times.

Step 1
State the general equation of a trigonometric function

$y = A \cos B(x - C) + D$

Step 2
Clearly state values of *A*, *D* and *B*

$A = \frac{\text{max value} - \text{min value}}{2} = \frac{50-35}{2} = 7.5$

$D = \text{max value} - A = 50 - 7.5 = 42.5$

The period is 28 × 2 = 56 minutes. i.e. It takes 28 minutes to get from a maximum to a minimum, therefore it will take 56 minutes to get from a minimum to a minimum.

$B = \frac{2\pi}{56} = \frac{\pi}{28}$

Step 3
Calculate *C* by substituting in a known (*x*, *y*) value

$y = A \cos B(x - C) + D$

$y = 7.5 \cos \frac{\pi}{28}(x - C) + 42.5$

A known (*x*, *y*) is (15, 50) from the problem

$50 = 7.5 \cos \frac{\pi}{28}(15 - C) + 42.5$

$C = 15$

Step 4
Write out the equation and check using the GRAPH function on the graphics calculator. If there is not a maximum at (15, 50) or a minimum at (43, 35), then check your calculations.

$y = 7.5 \cos \frac{\pi}{28}(x - 15) + 42.5$

b) When the creature reaches a temperature of 48°C or higher, it needs to feed. At what intervals will this creature need to feed? You can write this as a general equation.

Step 1
Set up equation to solve and use general solution to get values

$$48 = 7.5\cos\frac{\pi}{28}(x-15) + 42.5$$
$$5.5 = 7.5\cos\frac{\pi}{28}(x-15)$$
$$0.733 = \cos\frac{\pi}{28}(x-15)$$
$$0.747 = \frac{\pi}{28}(x-15) \qquad \alpha = 0.747$$

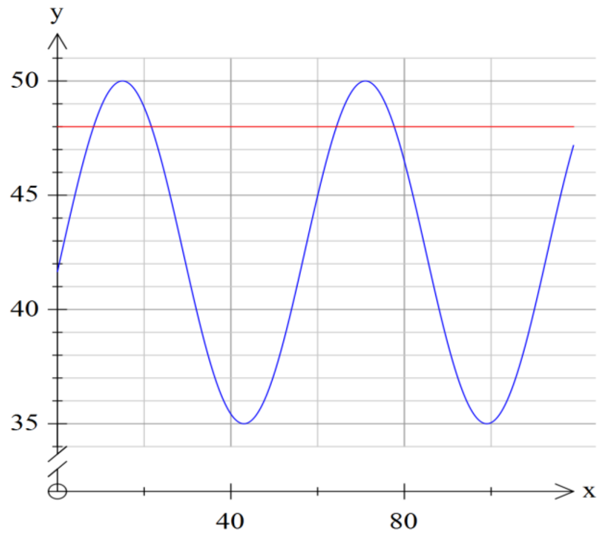
$$\frac{\pi}{28}(x-15) = 2n\pi \pm 0.747$$
$$x = 56n \pm 6.658 + 15$$

Step 2
Find the x-values for when the creature needs a feeding by letting $n = 0, 1, 2, \ldots$

$n = 0$ $\quad x = 56(0) - 6.658 + 15 = 8.342$
$\quad x = 56(0) + 6.658 + 15 = 21.652$
$n=1$ $\quad x = 56(1) - 6.658 + 15 = 64.342$
$\quad x = 56(1) + 6.658 + 15 = 77.658$
$n = 2$ $\quad x = 56(2) - 6.658 + 15 = 120.342$
$\quad x = 56(2) + 6.658 + 15 = 133.658$

Step 3
Sketch a graph to check the intervals at which feeding needs to occur.

Check that the solutions for $x = \ldots$ are also correct using the graphics calculator.

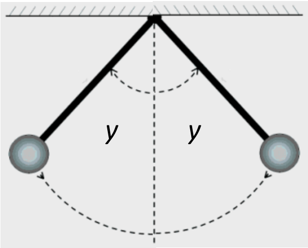


The parts of the graph that are above the line $y = 48$ is where the creature needs feeding.
This means between 8.342 minutes and 21.652 minutes, the creature will first need feeding. This cycle will repeat every 56 minutes, as this is the period of the function.

Exercise VIII: Applications of Trigonometric Graphs

The angle a swinging pendulum in a vacuum makes with a vertical line (in either direction) can be modelled by a trigonometric function.

- The pendulum is released at an angle of 40 degrees. This is the maximum angle.
- The minimum angle possible is zero
- The pendulum completes a swing (from left to right, and back to starting position) 4 times per second.



Use the above information to give the equation of the model, and therefore the angle the pendulum is at 0.2 seconds after the pendulum is released.

ONE

At a certain beach, there is a height marker 1m out from the foreshore. The day Geoff planned to go windsurfing, the water height at this point could be modelled using a trigonometric function.

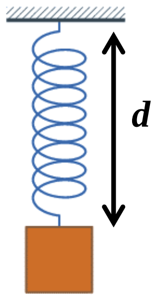
Geoff starts recording the height of the waves at 8 o'clock in the morning, when the waves are at a maximum of 2.75m. 6 hours later, the waves are at a minimum height of 0.25m.

Geoff does not like launching when the water at this point is over his neck, a height of 1.5m. If the waves maintain this trigonometric model the following day, at what time will Geoff have his launching opportunities during the day, and for how long will this last?

TWO

THREE

A block is attached to a spring. The spring is extended and released.



The distance, ***d*** centimeters, of the top of the block below the spring's attachment point ***t*** seconds after release can be modelled by a trigonometric equation

It takes the block 4 seconds to return to its starting point.
The closest it gets to its attachment point is 2.5 cm and the furthest is 8.5 cm.
Find the equation for *d* and use it to find when the block is first 4 cm from the attachment point.

FOUR

Two people are turning a skipping rope. The height of the rope handle (h) above the ground at t seconds after the rope starts to turn is modelled by a trigonometric equation. At the lowest point the handle is 66cm above the ground. It reaches a maximum height of 190c m above the ground. One complete turn takes 1.8 seconds.

When would the rope by 1.4m, or higher above the ground?

Practice Assessment: Liath the Kitten

Liath the Kitten has been given two new toys. Each toy is battery-operated and moves up and down in front of her.

The first toy has a feather attached to it. When not moving, the tip of the feather is resting 30cm off the ground. The feather is then pulled down 12 cm and then released causing the feather to oscillate with a period of 12 seconds.

The second toy has a ball attached to it. The toy reaches its maximum height of 62cm off the ground 4 seconds after it has been switched on. Its minimum distance from the ground is 20cm 7 seconds later.

Liath likes batting these toys, but unfortunately she is a very small kitten. Her reach is only 23cm from the ground.

- Write equations for the movement of both toys using sine and cosine functions.
- Find the general solution for when Liath can reach the first toy.
- Find the times when Liath can reach both toys at the same time.