Introduction

Ted grows artichokes and tomatoes. He is looking at planting options to maximise his income.

This activity requires you to use linear programming to model the constraints Ted has for his planting and to make recommendations so that he can maximise his income in the current year and in future years. You will present your findings as a written report, supported by graphs, equations and relevant calculations.

Task

Ted produces tomatoes for a local factory and sells artichokes at a farmers' market. Artichokes are very labour-intensive and Ted is looking at his planting options in order to maximise his income.

Using the constraints outlined in the Resource Sheet, write a report making recommendations as to how many hectares of tomatoes and artichokes Ted should plant to maximise his income in the current year and in future years.

As you write your report take care to clearly communicate your findings using appropriate mathematical statements. Include graphs, equations, and relevant calculations.

Resource Sheet

Constraints

Artichokes require 20 hours of labour per hectare, tomatoes require 10 hours per hectare. Ted has 1200 hours of labour available for the two crops.

Ted has 90 hectares available for planting altogether.

To keep his contract with the factory, Ted must plant at least 30 hectares of tomatoes. He feels that planting anything less than 10 hectares of artichokes would make travel to the farmers' market uneconomical.

Income predictions

For the current year, Ted expects to receive $10,000 per hectare for his tomatoes and $25,000 per hectare for his artichokes.

Tasks

1. Find how many hectares of each crop Ted should plant to maximise his income.

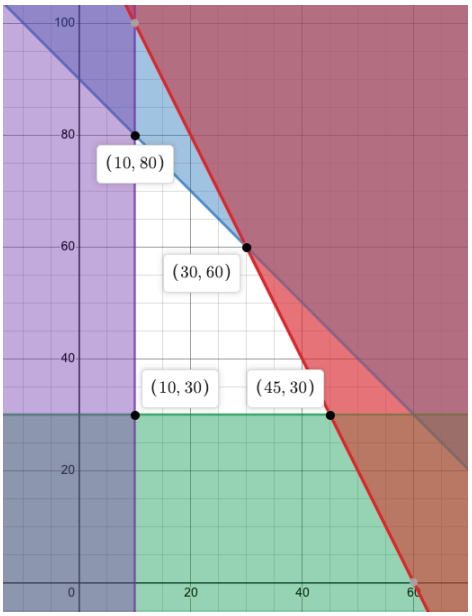
2. The future value of tomatoes and artichokes is unknown. However, payments per hectare of tomatoes and hectare of artichokes are forecast to be in a ratio of 1:2. Find how many hectares of each crop Ted should now plant to maximise his income.

3. Ted's mate Rodney believes the future value of artichokes is expected to drop so in the future Ted would like to produce tomatoes and artichokes at a ratio of between 5:4 and 9:4. Use the initial income expression above to maximise Ted's income.
   To keep things simple for Ted, please give recommendations for hectares as whole numbers.

Answers

1. Let x = hectares of artichokes
   Let y = hectares of tomatoes

   $20x + 10y \leq 1200$
   $x + y \leq 90$
   $y \geq 30$
   $x \geq 10$



   Income will be at a maximum at one of the corners of the feasible region (critical points).
   Critical points are (10,80), (30,60), (45,30), (10,30)

   Income = 25000x + 10000y

   Income at (10,80) = $1,050,000
   Income at (30,60) = $1,350,000
   Income at (45,30) = $1,425,000
   Income at (10,30) = $550,000

   So Ted should plant 45 hectares of artichokes and 30 hectares of tomatoes giving a maximum income of $1,425,000

2. Future income will be determined by the equation:
   Income = 2nx + ny
   where n = any integer

   eg. If n = 10000,
   Income = 20000x + 10000y
   Income at (10,80) = $1,000,000
   Income at (30,60) = $1,200,000
   Income at (45,30) = $1,200,000
   Income at (10,30) = $ 500,000

   So in this case, growing 20 hectares of artichokes and 60 hectares of tomatoes, or growing 45 hectares of artichokes and 30 hectares of tomatoes will both give a

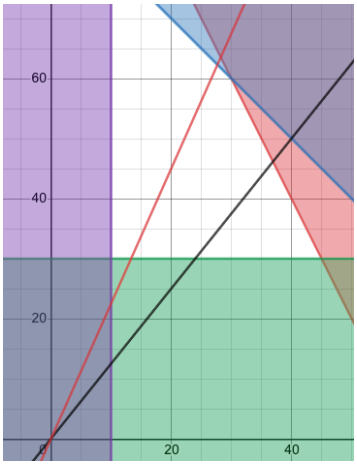
maximum income of $1.2 million

3. Ratio of artichokes:tomatoes, or x:y is between 4:5 and 4:9

   x:y
   4:5
   means 5x=4y or rearranging, y = 5/4 x

   x:y
   4:9
   means 9x=4y or rearranging, y = 9/4 x

   Plotting the lines x = ⅘ y and x = 4/9 y in desmos gives:



   and we are looking for the maximum income that lies between these two lines.

   Income = 25000x + 10000y

   Critical points:
   (27.692, 62.308):
   Check:
   (27,62) - no
   (27,63) - no
   (28,62) - yes in feasible region, Income = $1,320,000
   (28,63) - no

   (30, 60), Income = $1,350,000

   (36.923,46.154):
   Check:
   (36,46) - yes in feasible region, Income = $1,360,000
   (36,47) - yes in feasible region, Income = $1,370,000
   (37,46) - no, outside desired ratios
   (37,47) - no, outside feasible region

   (24,30) will clearly not maximise income
   (13.333, 30) will clearly not maximise income

   So to maximise income at Ted's preferred ratios, he should plant 36 hectares of artichokes and 47 hectares of tomatoes, giving an income of $1.37 million.