Algebra Past Papers - Modelling/Word Problems- Answers

2023 Question 3d-e.

(d)	Equation $y = a(x+1.25)(x-1.25)$ $y = a(x^2 - 1.5625)$ $x = 0,\ y = -3$ $-3 = -1.5625a$ $a = 1.92$ $y = 1.92(x+1.25)(x-1.25)$ $x = 1.1,\ y = -0.678$ No, boat will not float. OR Using vertex form: $y = a(x-h)^2 + k$ $y = ax^2$ $3 = a(1.25)^2$ $a = 1.92$ $y = 1.92(1.1)^2$ $y = 2.3232$ The edge of the canal is only 0.6768 m below the water surface, so the boat won't float. Accept other correct variations of these approaches.	• General equation, fitting the context, formed in any correct format.	• Coefficient of x^2 found..	• Depth of canal at appropriate width, or width of canal at 1m depth, calculated. AND Statement that boat will not float or similar comment.
(e)	$2\pi rh + 2\pi r^2 = 8rh + 4r^2$ $\pi rh + \pi r^2 = 4rh + 2r^2$ $\pi h + \pi r = 4h + 2r$ $\pi h - 4h = 2r - \pi r$ $h(\pi - 4) = r(2-\pi)$ $h = r\dfrac{(2-\pi)}{(\pi-4)}$ $= \dfrac{r(\pi-2)}{(4-\pi)}$ i.e. $h = 1.33r$		• Sets up initial equation. OR gives r correctly in terms of h. box is incorrect 2 OR If surface area of **and** SA $= Ar + Bhr$, where B ≠ 0, accept consistent, possibly unsimplified, answer of $h = \dfrac{r\left(\pi - \frac{A}{2}\right)}{\left(\frac{B}{2} - \pi\right)}$	T: Correct expression but not simplified. TT: Correct expression.

2022 Question 1c-d.

(c)(i)	Sum of orange corners: A + A + 24 = 2A + 24 [A + B] Sum of blue corners: A + 21 + A + 3 = 2A + 24 [(A+3) + (B – 3)] Therefore sum of orange corners = sum of blue corners, no matter where you start the square.	Correct algebraic evidence but no conclusion.	Two sums compared and conclusion explicitly drawn.	
(ii)	Product of orange corners: $A(A + 24) = A^2 + 24A$ Product of blue corners: $(A + 21)(A + 3) = A^2 + 24A + 63$ If these products are equal: $A^2 + 24A + 63 = A^2 + 24A$ ** So 63 = 0 Which is impossible. Or a statement that 63 cannot equal zero. OR An argument based on the orange corners being A and B, and the blue corners being A + 3 and B – 3, leading to 3B – 3A – 9 = 0 B – A = 3 ## This cannot true if B is on a different row, and, as this is not true, the products cannot be equal. [or equivalent arguments with different valid expressions for the corners]		Correct algebraic evidence up to line **. OR Simplified relationship between A and B (line ##) but no conclusion	Correct and complete algebraic reasoning. OR Correct algebraic evidence with conclusion.

(d)	For a rectangle M wide and N tall: Sum of orange corners: A + [A + (M – 1) + 7(N – 1)] = 2A + M + 7N – 8 Sum of blue corners: [A + 7(N – 1)] + [A + (M – 1)] =2A + M + 7N – 8 Both sums have the same expression so are always equal. Accept alternative approaches that are valid arguments.		Reasoning valid but M and / or N used for the corners. OR Correct algebraic evidence but no conclusion.	Correct and complete reasoning with conclusion stated.

2021 Question 3a-d.

(a)	Volume of cuboid = p^3 in simplest form.	Correct answer.		
(b)	$(p-4)(p+5)(p-3)$ $=(p-4)(p^2+2p-15)$ $=p^3-2p^2-23p+60$	Correct simplified expression.		
(c)	Volume of cuboid $=p(p+a)(p-a)$ $=p^3-pa^2$ If this is the same as the volume of the cube, $p^3-pa^2=p^3$ so $pa^2=0$ Thus $a=0$ is the only solution (since $p \neq 0$). OR that there are no [non-trivial] solutions. Hence the cuboid never has the same volume as the cube unless you leave the sides unchanged. Or equivalent.	Correct working to the point of equating the two volumes.	Makes correct conclusion with valid working and reasoning.	
(d)(i)	SA of cuboid $=2[(p-a)(p+a)+10(p-a)+10(p+a)]$ $=2[p^2-a^2+20p]$ Thus $2p^2-2a^2+40p=6p^2$ $0=4p^2-40p+2a^2$ $0=2p^2-20p+a^2$	Finds expression for SA of cuboid and expands it (accept no use of factor of 2 for all sides).	Derives given equation clearly.	
(d)(ii)	For there to be solutions, $400-8a^2>0$ $50>a^2$ Hence $0<a<7.1$. If $a=7$, we solve $2c^2-20c+49=0$ and obtain $p=5.707$ or $p=4.293$. However, one of the sides of the cuboid is $p-a$, meaning that it will be negative with either value of p obtained here. If $a=6$, we solve $2p^2-20p+36=0$ and obtain $p=7.646$ or $p=2.354$. The larger of these 2 values is a valid solution so the largest possible whole number value of a is 6. The cube has sides of 7.646 cm, and the cuboid is $1.646 \times 13.646 \times 10$ cm.	Evaluates discriminant.	Obtains inequality (or implies one) for a or a^2.	T1: Obtains $a=7$ and dimensions of 5.707 for the cube but does not go on from there. T2: Obtains correct dimensions for the cube and the cuboid.

2020 Question 3c.

(c)(i)	Turnover = $(2d + 5)(101 - 3d) = 445$ $-6d^2 + 187d + 60 = 0$ Either $d = -0.3176$ or $d = 31.48$ (4sf) d needs to be both positive and whole, so neither solution is valid, which means that the turnover is never \$445.	Forms the correct equation for turnover.	Finds the values of d.	T1: Gives a valid explanation as to why the turnover is never \$445.
(c)(ii)	$(2d + 5)(101 - 3d) = k$ $-6d^2 + 187d + (505 - k) = 0$ $d = \frac{-187 \pm \sqrt{187^2 - 4(-6)\leftarrow(505-k)}}{2(-6)}$ $d = \frac{187 \pm \sqrt{47089 - 24k}}{12}$ 1. Discriminant needs to be positive (so $k < 1962.04$) 2. d is rational so $47089 - 24k$ must be a square number 3. d is an integer, so $187 \pm \sqrt{47089 - 24k}$ must be divisible by 12 4. d is positive, so $187 \pm \sqrt{47089 - 24k}$ must be positive.	Rearrangement of equation set to 0.	Finds a simplified expression for d.	T1: Makes ONE of the listed conclusions. T2: Makes TWO of the listed conclusions.

2019 Question 2d.

(d)	Small rectangle: Area = $y^2 - 8y = 9$ $y^2 - 8y - 9 = (y - 9)(y + 1) = 0 \Rightarrow y = 9$ Since $x = 2y - 6$ Large rectangle: Area = $(2y - 6)(2y - 10)$ $= 4y^2 - 32y + 60$ Hence Area = 96 cm^2	y found.		Area found.

2018 Question 1e.

(e)	$h = ¼ (w + 60) = ¼w + 15$ (or $w = 4h - 60$) $A = 60w + 2 \times wh + 2 \times 60h$ $= 60w + 2wh + 120h$ $7400 = 60w + 2w (¼w + 15)$ $+ 120 (¼w + 15)$ $\Rightarrow ½w^2 + 120w - 5600 = 0$ (or $8h^2 + 240h - 11\,000 = 0$) $w = 40, -280$ (or $h = 25, -55$) Hence $h = ¼ (40 + 60) = 25$ cm	Expression for height or area formed.	Quadratic formed.	Height found.

2018 Question 2d.

(d)	$k = \frac{2.43}{(1.8)^2} = 0.75$ $h = 0.75x(3.6 - x) = 2.7x - 0.75x^2$ $0.75x^2 - 2.7x + 0.5 = 0$ $x = 3.4041, 0.1958$ Length of rail = 3.4041 – 0.1958 = 3.208 metres	Finds k.	Forms a quadratic.	Length of rail found.

2017 Question 1c-d.

(c)	$(2n+6)^2 - (n-2)^2 = 200$ $4n^2 + 24n + 36 - n^2 + 4n - 4 = 200$ $3n^2 + 28n - 168 = 0$ $n = 4.1525$ OR -13.49 Width is $\frac{1}{2}(2n+6-n+2) = \frac{n}{2} + 4$ $= 6.1$ cm (6.07625)	Sets up a quadratic OR finds expression for width in terms of n.	Both solutions of quadratic found.	Correct answer.
(d)	Let x be the number of people who went on the trip. $\frac{560}{x} - \frac{560}{x+3} = 1.5$ $1.5x^2 + 4.5x - 1680 = 0$ $x = 32$ or -35 So 32 students went on the trip.	Sets up equation.	Quadratic equation formed.	Correct solution with positive answer.

2016 Question 2e.

(e)	Let the sides of the triangle be $3y$, $4y$, and $5y$ for some real positive number y. Area of triangle is $\frac{1}{2} \times 3y \times 4y = 6y^2$ Path has width 1. So path area is $12y + \pi$ $2(12y + \pi) - 6y^2 = 2\pi$ $24y + 2\pi - 6y^2 - 2\pi = 0$ $24y - 6y^2 = 0$ $6y(4 - y) = 0$ So $y = 4$ (as can't be 0) and length of longest side of triangle is $5 \times 4 = 20$ m. (Longest side 22.35m accepted as an alternative interpretation.)	Quadratic established.	Quadratic solved for y, or consistently solved from incorrect quadratic.	Correctly solved and dimensions given.

2015 Question 2d-e.

(d)(i)	Let x be the length and w the width. Then the perimeter is $2x + 2w$. Area $xw = 50$ So $w = \frac{50}{x}$ or $2w = \frac{100}{x}$ So perimeter = $2x + \frac{100}{x}$	Shows relationship.		
(d)(ii)	$2x + \frac{100}{x} = 33$ $2x^2 - 33x + 100 = 0$ $(2x-25)(x-4) = 0$ $x = 12.5$ or $x = 4$ m So the dimensions of the garden are 4 m and 12.5 m.	Forms a quadratic equation	Solved for x, or consistently solved from incorrect quadratic.	Correctly solved and dimensions given.

(e)	David's speed is x km / h Sione's speed is $(x + 4)$ km / h Difference in time is half an hour. $0.5 = \frac{150}{x} - \frac{150}{x+4}$ $0.5 = \frac{150(x+4) - 150x}{x(x+4)}$ $0.5x(x+4) = 600$ $x^2 + 4x - 1200 = 0$ $x = 32.70$ km / hr		Sets up equation correctly and solves with an error.	Correctly sets up equation and solves correctly.

2014 Question 2c.

(c)(i)	$rx^2 - tx - h = 0$ $x = \frac{t \pm \sqrt{t^2 + 4rh}}{2r}$		Answer with ± before surd.	
(ii)	$h = ax(x-12)$ when $x = 6$, $h = 6$ $6 = 6a \times -6$ $a = -\frac{1}{6}$ $h = -\frac{1}{6}x(x-12)$ Or $y = -\frac{1}{6}x^2 + 2x$	General form of equation and recognition of point (6,6).	Correct equation.	

2014 Question 2c cont.

(iii)	$h = -\frac{1}{6}x(x-12) = 1.9$ $-x^2 + 12x - 6 \times 1.9 = 0$ $x = 1.04,\ 10.96$ width of lane 4.955 m		Solved for height of 1.9 m.	Correct width of lane found.

2012 Question 2e.

(e)	Equation $d = a(x+8)(x-8)$ $d = a(x^2 - 64)$ $d = 0 \quad 64a = 16$ $a = ¼$ $d = ¼\,(x+8)(x-8)$ Width of 12, $x = \pm 6$ $¼ \times 14 \times 2 =\ 7$ m	General equation formed in any correct format.	*a* calculated and equation formed. Depth = –7m	Problem solved.