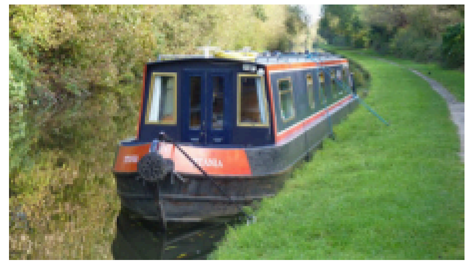
Algebra Collated Past Papers - Modelling/Word Problems

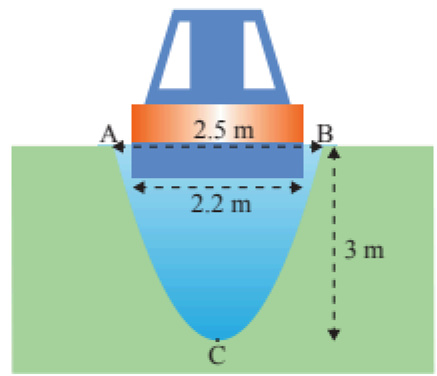
2023 Question 3d-e.

(d) The diagram below shows the cross-section of a canal and narrowboat floating in the canal.

- The surface of the water (between points A and B) measures 2.5 m across.
- The canal is 3 m deep, at the deepest point C.
- The cross-section of the canal can be modelled as a quadratic curve ACB.
- The cross-section of a narrowboat on the canal can be modelled as a rectangle with a width of 2.2 m.
- The narrowboat must maintain a constant depth below the water of 1 m in order to float.

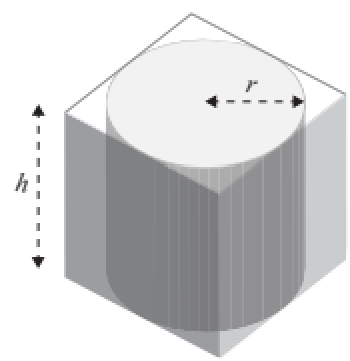


Source: www.pxfuel.com/en/free-photo-xerdj



Will the narrowboat be able to float in this canal?

(e) An open box (i.e. with a base but no lid) has been designed to tightly fit a cylindrical candle. The surface area of the five surfaces of the box is equal to the total surface area of the candle.



Write an expression for the height, h, in terms of the radius, r, and π.

(Surface area of a cylinder = $2\pi r^2 + 2\pi rh$.)

2022 Question 1c-d.

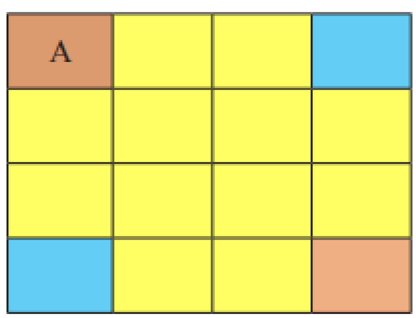
(c) A calendar can be presented in the following way, where each day is given a number from 1 to 365. This is the beginning of a year's calendar:

M	T	W	TH	F	SA	SU
				1	2	3
4	5	6	7	8	9	10
11	12	13	14	15	16	17
18	19	20	21	22	23	24
25	26	27	28	29	30	31
32	33	34	35	36	37	38
39	40	41	42	43	44	45
46	47	48	49	50	51	52
53	54	55	56	57	58	59
60	61	62	63	64	65	66
67	68	69	70	71	72	73

Jo draws a 4-by-4 square on the calendar to check a claim that she heard:

"*the sums of the diagonally opposite corners are always the same, no matter where you make your square*". In other words, when you add the numbers in the orange corners, it is the same as when you add the numbers in the blue corners.

Jo wonders if the claim will still be true no matter where she starts the square, so she begins an investigation using algebra:



(i) Use algebra to prove that, no matter where the 4-by-4 square is drawn on the calendar, **the sum of the orange corners must be the same as the sum of the blue corners**.

(ii) Jo wonders about the **products** of the diagonally opposite corners: could they be the same?

Use algebra to prove that it is **not** possible to draw any 4-by-4 square for which the products of the diagonally opposite corners are the same.

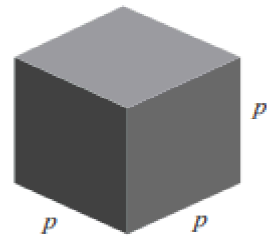
(d) Will it always be true that the **sum** of the orange corners must be the same as the sum of the blue corners, regardless of the size or shape of the rectangle Jo draws?

Use algebra to support your answer by considering an m-by-n rectangle drawn on the calendar below (where m and n are whole numbers greater than 1, and $m \neq n$).

You may wish to draw a diagram on the calendar, or beside it, to help explain your reasoning.

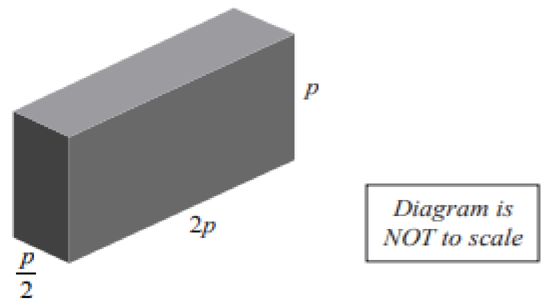
2021 Question 3a-d.

Consider a cube with sides of p cm (where $p \neq 0$). The volume of the cube would be p^3 cm^3, and the surface area of the cube would be $6p^2$ cm^2.



Junyang wonders if it is possible to change the dimensions of the cube to make a cuboid that still has the same volume.

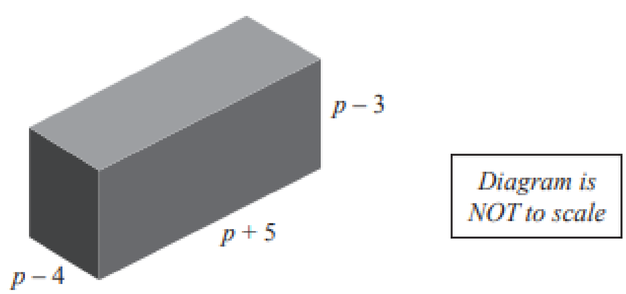
(a) First, he tries doubling the length, halving the width, and keeping the height as p, as sketched below.



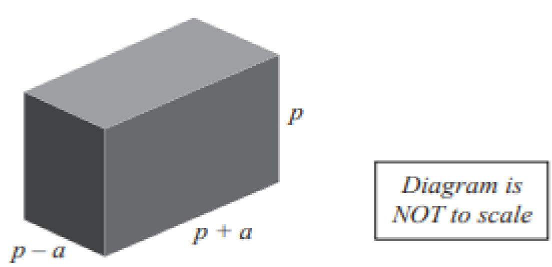
Using algebra, find the volume of this cuboid.

(b) The volume of the cuboid below is given by the expression: $(p - 4)(p + 5)(p - 3)$.

Expand and simplify this expression.



(c) Next, Junyang tries adding an amount, a, to the length and then taking off the same amount from the width, keeping the height the same (see below).



Are there any values of a for which the volume of the cuboid will be the same as the volume of the cube?

(d) Junyang realises that the **surface area** of the cuboid will not be the same as the surface area of the cube unless he also changes the height. He decides to make the height of the cuboid 10 cm.

He wants to find out which value of p would result in the cube having the same surface area as the cuboid. To do this, he needs to form and solve an equation for p.

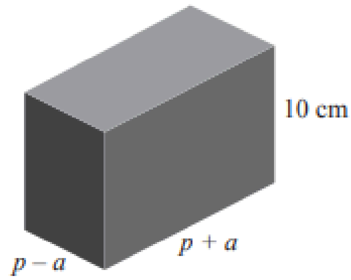


Diagram is NOT to scale

(i) If the surface area of the cube is the same as the surface area of the cuboid, show that $2p^2 - 20p + a^2 = 0$.

Remember that the surface area of the cube is $6p^2$ cm^2.

(ii) As mentioned in part (i), if the surface area of the cube is the same as the surface area of the cuboid, then $2p^2 - 20p + a^2 = 0$.

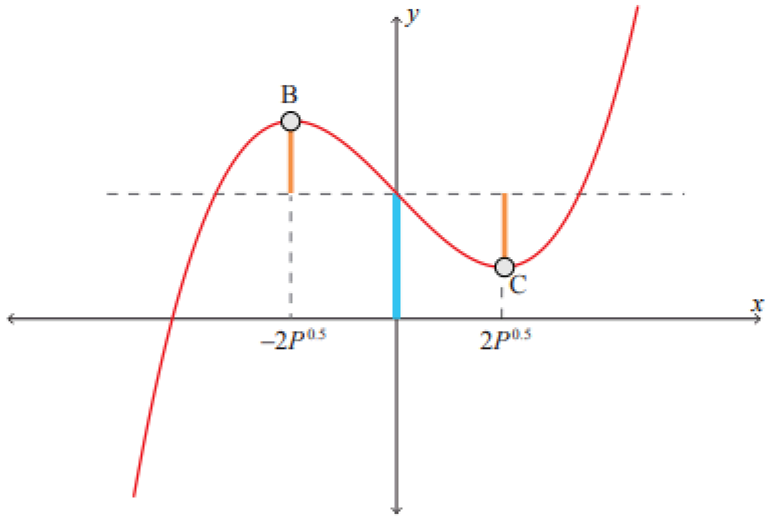
By using the discriminant (Δ), find the largest possible **whole** number value that a could take in this context.

Use this value of a to find the dimensions of both the cube and the cuboid.

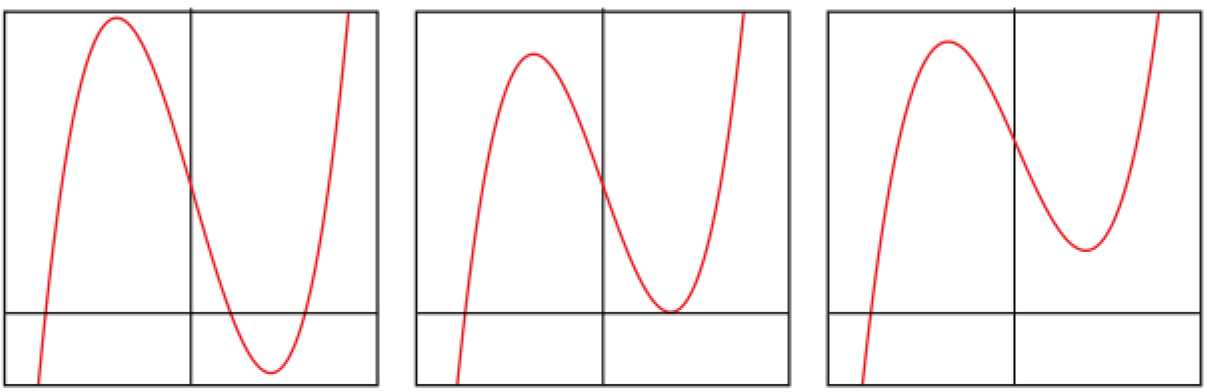
Explain your reasoning clearly.

2020 Question 1c.

(iii) Consider again the curve with the equation $y = x^3 - 12Px + R$. As the values of P and R vary, the shape of the curve changes, and the lengths of the orange lines and of the central blue line (below) vary. However, by symmetry, the two orange lines remain the same length as each other.



Some examples of the graphs obtained from various values of P and R are illustrated below.



For some combinations of P and R, the curve can intersect the x-axis three times. This will happen if each orange line is longer than the blue line.

2020 Question 3c.

(c) Zahra sells zips. Zahra notices that the higher the price of a zip, the fewer zips are sold. As an experiment, Zahra increases the price of a zip by \$2 each day (starting at \$7) and keeps a record of how many zips are sold each day. She does this for 6 days and finds that the number of zips sold each day started at 98 and is dropping by 3 each day.

The total amount of money Zahra received each day for zips, the turnover, is also recorded in the table below.

Day, d	**1**	**2**	**3**	**4**	**5**	**6**
Price of a zip (\$) = $2d + 5$	7	9	11	13	15	17
Number of zips sold = $101 - 3d$	98	95	92	89	86	83
Turnover (\$)	686	855	1012	1157	1290	1411

(i) If all the patterns continue to be valid, is there any day on which the turnover is exactly \$445? Use algebra to justify your answer and explain your conclusions.

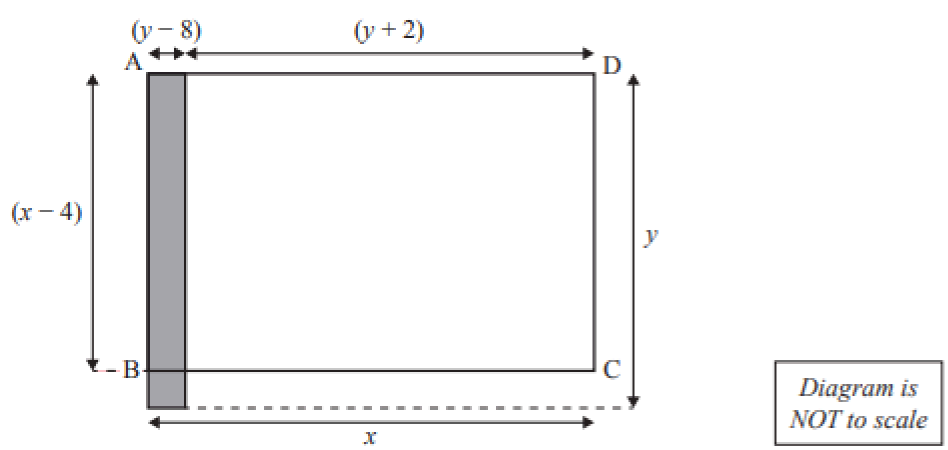
(ii) Zahra realises that not every whole number is a possible turnover value for a given day.

Using algebra, find at least three conditions a whole number k must satisfy for it to be a possible turnover for a given day.

2019 Question 2d.

(d) The shape below is divided into rectangles. All measurements are in cm.

This diagram has been corrected from that used in the examination.



The shaded rectangle has an area of 9 cm^2.

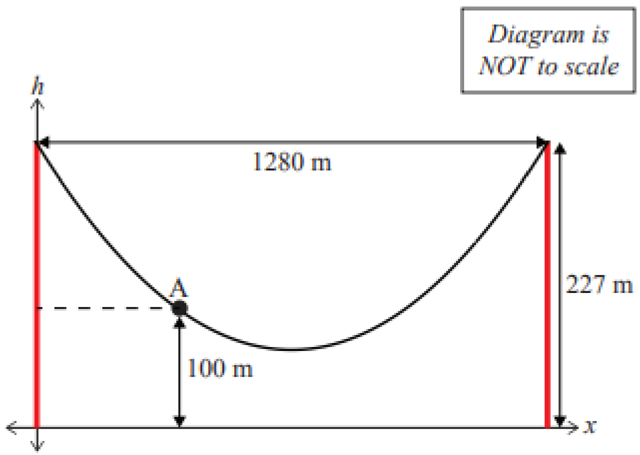
Find the area of the rectangle ABCD.

2019 Question 3e.

(e) The Golden Gate Bridge in San Francisco has two towers.

The height h in metres of the suspension cables above the mean water level, at a horizontal distance x metres from the base of the left tower, can be modelled by the function $h = k(x - 640)^2 + 67$.

At the mid-point between the two towers, the suspension cables are 67 metres above the mean water level. The distance between the towers is 1280 metres and the towers are 227 metres tall, measured from the mean water level.



Rich Niewiroski Jr (https://commons.wikimedia.org/wiki/File:GoldenGateBridge-001.jpg), CC BY 2.5.

An anemometer (shown as A in the left-hand diagram above) to measure wind speed is placed on a cable at a height of 100 metres above the mean water level.

Find the horizontal distance of the anemometer from the left tower.

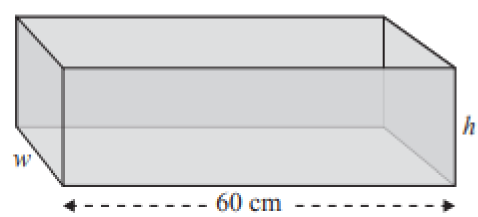
2018 Question 1e.

(e) A rectangular box has no lid.

The length of the base is 60 cm.

Its height is one quarter of the sum of its width and length.

The total area of the base **and** the four sides of the box is 7400 cm^2.

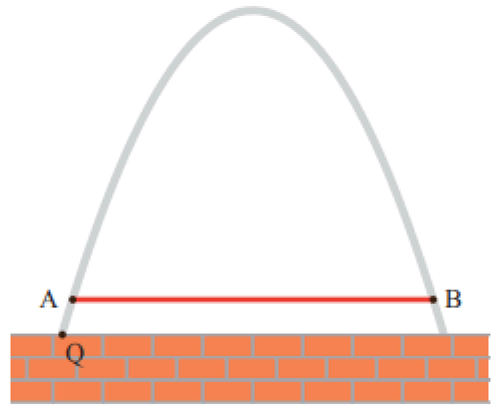


Find the height of the box.

2018 Question 2d.

(d) An equestrian jump has a parabolic arch mounted on a wall. Horses and riders jump through the arch.

Source: http://luxequestrian.com/slideshow/incredible-jumps-brody-robertson



The arch rises 2.43 metres **above the wall**.

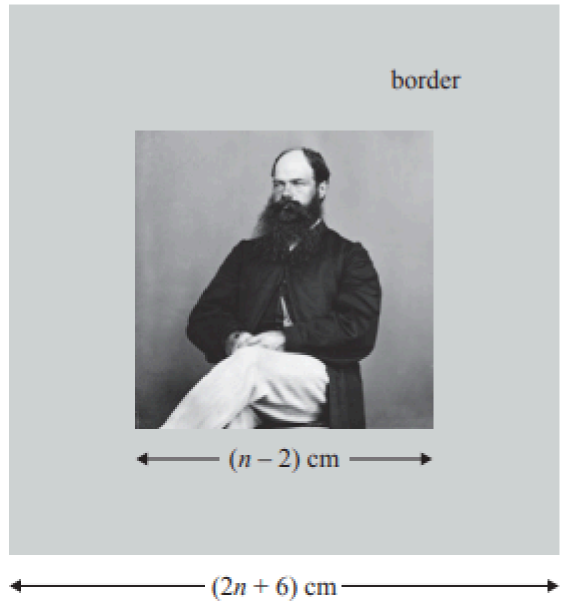
The arch can be modelled by a function of the form $h(x) = kx(3.6 - x)$, where k is a constant, h metres is the height above the wall, and x metres is the horizontal distance from Q.

A rail AB can be placed above the wall and attached at each end to the arch. For one competition, the rail is placed 0.5 metres above the wall.

How long is the rail AB?

2017 Question 1c-d.

(c) David has mounted a square photo on a square piece of card as shown below.



The border around the photo is of constant width.

The photo has sides of length $(n - 2)$ cm while the card has sides of $(2n + 6)$ cm.

If the total area of the border is 200 cm^2, find the width of the border.

(d) A teacher has hired a school bus for \$560 for a day trip with students.

The cost of hiring the bus is to be shared equally between the students.

At the last moment, three of the students were unable to go.

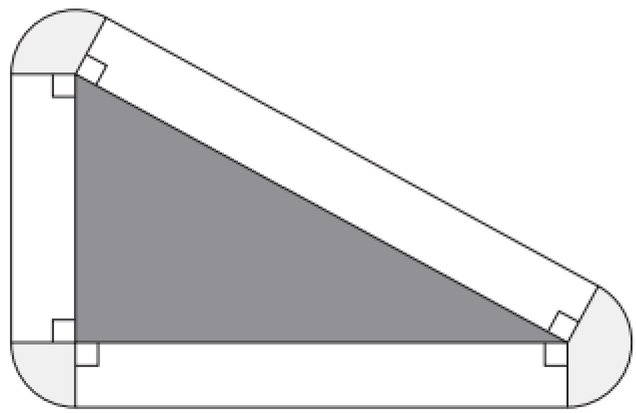
As a result, the cost to each of those who did go was increased by \$1.50.

How many students finally went on the trip?

Justify your answer.

2016 Question 2e.

(e) The diagram below shows a triangular garden with a path around it.



The triangular garden has sides with lengths in the ratio 3:4:5.

The path is 1 m wide.

At each corner of the garden, the path is a sector (part) of a circle with a radius of 1 m.

The difference between **twice** the **total** area of the path and the area of the garden is 2π m^2.

Find the length of the longest side of the garden.

(Area of circle = πr^2)

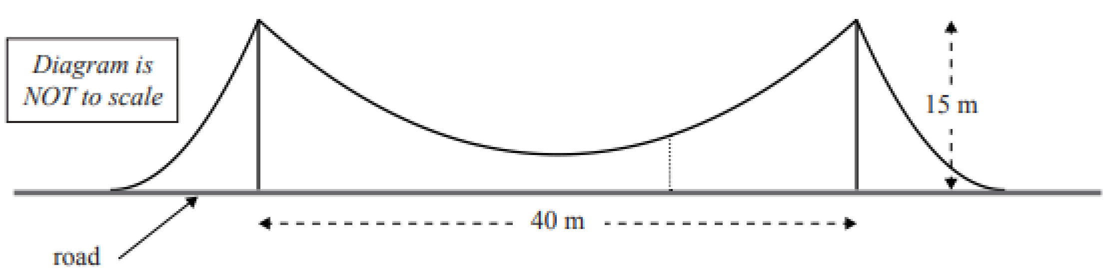
2016 Question 3e.

(e) A symmetrical bridge has its central cable in the shape of a parabola, as shown in the diagram below.

The towers supporting the cable are each 15 m high and 40 m apart.

At the point midway between the towers, the height of the cable above the road is 3 m.

A vertical post (shown dotted in the diagram) is placed 10 m from the centre of the bridge and just touches the cable.



(i) Use algebra to show that the post is 6 m high.

(ii) The length of the bridge AB is 60 m.

The outside cables are also parabolic and symmetrical in shape, and touch the road at their vertices A and B.

Find the distance, CD, between the two parabolas at a height of 6 m above the road (the distance CD is shown in the diagram).

2015 Question 2d-e.

(d) Talia used timber to form the exterior sides of her rectangular garden. The length of the garden is x metres, and its area is 50 m^2.

(i) Show that the perimeter of the garden is given by $2x + \frac{100}{x}$

(ii) If she uses 33 m of timber to build the sides, find the dimensions of the garden.

(e) David and Sione are competing in a cycle race of 150 km.

Sione cycles on average 4 km per hour faster than David, and finishes half an hour earlier than David.

Find David's average speed.

You MUST use algebra to solve this problem. (Hint: average speed $= \frac{distance}{time}$*)*

2014 Question 2c.

(c) (i) The height h metres of a tunnel is modelled by a function of the form

$$h = rx^2 - tx$$

where r and t are constants.

Make x, the distance in metres from the left side of the tunnel, the subject of the equation.

(ii) The shape of the tunnel can be modelled by a parabola.

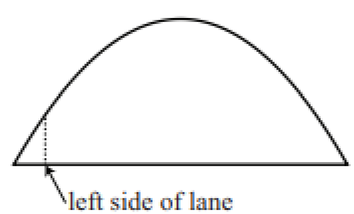
The maximum height of the tunnel is 6 m, and at ground level its width is 12 m.

Find the equation of the parabola.

(iii) There are two lanes of equal width through the tunnel.

The outside edge of each lane is marked by a line so that a car of height 1.8 m would have a minimum clearance of 0.1 m vertically from the top of the car to the tunnel roof.

(Ignore the width of the line.)



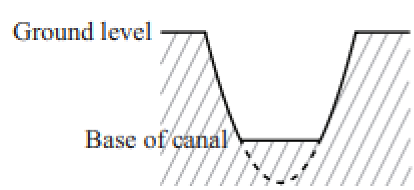
Find the width of each lane.

2012 Question 2e.

(e) The width of a canal at ground level is 16 m.

The **sides** of the canal can be modelled by a quadratic expression that would give a maximum depth of 16 m.

However, the base of the canal is **flat** and has a width of 12 m.



What is the actual depth of the canal?