



Algebra Past Papers - Algebraic Manipulation - Answers

2023 Question 1a-b.

ONE (a)(i)	$\frac{2}{n}$	<ul style="list-style-type: none"> Correct simplified expression with positive index, as given. 		
(ii)	$\left(\frac{n^3}{16n^6}\right)^{-0.5} = \sqrt{16n^3}$ $= 4n^{\frac{3}{2}}$	<ul style="list-style-type: none"> Correct interpretation of negative power. OR Interprets power of 0.5 as square root. OR Obtains unsimplified equivalent fraction or numerical coefficients such as 2^2 or $\frac{1}{0.25}$. 		
(b)	$\frac{2(10x^2 - 11x + 3)}{x(5x - 3)} = \frac{2(2x - 1)(5x - 3)}{x(5x - 3)}$ $= \frac{2(2x - 1)}{x}$ $= 4 - \frac{2}{x}$ <p>$A = 4$ and $B = -2$</p>	<ul style="list-style-type: none"> Correct factorisation of numerator. 	<ul style="list-style-type: none"> Correct values for A and B (if not explicitly stated, line 3 is required). 	

2023 Question 3a.

THREE (a)(i)	$(5x - 2)(x + 3) = 0$ $x = \frac{2}{5}$ or -3	<ul style="list-style-type: none"> Both values correct. 		
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2023 Question 3c.

(c)	$\frac{x^2 + 2x + k}{(x + 5)(x + 2)} = \frac{x - 3}{x + 2}$ $x^2 + 2x + k = (x - 3)(x + 5)$ $x^2 + 2x + k = x^2 + 2x - 15$ <p>Therefore, $k = -15$</p> <p>Or equivalent approach.</p>	<ul style="list-style-type: none"> Makes progress towards solution by eliminating denominators, or equivalent. 	<ul style="list-style-type: none"> Value found. 	
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2022 Question 1a-b.

ONE (a)	$\frac{2x-3}{x+4} = 3$ $2x-3 = 3x+12$ $x = -15$	Correct solution.		
(b)(i)	$6x^3y - 15x^2\sqrt{y} = 3x^2\sqrt{y}(2x\sqrt{y} - 5)$ Accept $3x^2y\left(2x - \frac{5}{\sqrt{y}}\right)$	Obtains $3x^2(2xy - 5\sqrt{y})$	Correct expression.	
(ii)	$\frac{6x^2 - x - 12}{3x^2 - 5x - 12} = \frac{(2x-3)(3x+4)}{(3x+4)(x-3)}$ $= \frac{2x-3}{x-3}$ Don't penalise hashing a correct answer	Correct simplified fraction.		

2022 Question 2c.

(c)	$\sqrt{2x+3} = 3x$ $2x+3 = 9x^2$ $9x^2 - 2x - 3 = 0$ $x = 0.6991 \text{ or } x = -0.4768 \text{ (4sf)}$	Obtains correct quadratic.	Obtains both correct solutions.	
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2022 Question 3a.

THREE (a)(i)	$\sqrt{49y^{36}} = 7y^{18}$	Correct response.		
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2021 Question 1a-b.

(a)(i)	$27y^5$	Correct expression.		
(a)(ii)	$2y^9$	Correct expression.		
(b)	Deduce that $(3x+2)(x-4) = 0$ OR $\left(x + \frac{2}{3}\right)(x-4) = 0$ so Original equation was $3x^2 - 10x - 8 = 0$	Correct equation but coefficients not whole numbers OR incorrect relevant factorised form but consistent expanded form.	Correct equation.	

2020 Question 1a-c.

(a)	$(6x - 5)(x + 3)$	Correctly factorised.		
(b)	$f(x) = x^2 + 10x + 22$ $f(x) = (x + 5)^2 - 3$	Square completed correctly.		
(c)(i)	Substitute $x = 4, y = 40$: $40 = 4^3 - 12P \times 4 + R$ $40 = 64 - 48P + R$ $48P = 24 + R$ Rearrange to get $P = \frac{24 + R}{48}$	Substitute correctly.	Find an equivalent expression for P in terms of R .	
(c)(ii)	$3x^2 = 12P$ $x^2 = 4P$ $x = \pm\sqrt{4P}$ $x = \pm 2\sqrt{P}$ $x = \pm 2P^{0.5}$ However, point B has a negative x -value, so $x = -2P^{0.5}$	Correctly solves the equation to the point where $x = 2P^{0.5}$ OR $x = 2\sqrt{P}$ OR $x = \pm\sqrt{4P}$ (\pm required)	Finds $x = \pm 2P^{0.5}$	T1: Correct working and mathematical statements including an explanation for only using the negative value.

2020 Question 2c.

(c)	$\frac{(5x+4)(2x+1) - (3x-4)(x+4)}{(x+4)(2x+1)} = 2$ $\frac{10x^2 + 13x + 4 - [3x^2 + 8x - 16]}{(x+4)(2x+1)} = 2$ $\frac{7x^2 + 5x + 20}{2x^2 + 9x + 4} = 2$ $7x^2 + 5x + 20 = 4x^2 + 18x + 8$ $3x^2 - 13x + 12 = 0$ $(3x - 4)(x - 3) = 0$ Either $x = \frac{4}{3}$ or $x = 3$ OR $5x + 4 - \frac{(x+4)(3x-4)}{2x+1} = 2(x+4)$ $(5x+4)(2x+1) - (x+4)(3x-4) = 2(x+4)(2x+1)$ $7x^2 + 5x + 20 = 4x^2 + 18x + 8$ $3x^2 - 13x + 12 = 0$ $(3x - 4)(x - 3) = 0$ Either $x = \frac{4}{3}$ or $x = 3$	Begins to handle denominators in a correct way (adding the fractions using a common denominator or multiplying through by one denominator).	Correct solutions.	
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2020 Question 3b.

(b)(i)	$x = W^{\frac{5}{2}} - 2 = \sqrt{W^5} - 2$	Correct expression.		
(b)(ii)	$(x+2)^{\frac{2}{5}} < 20$ $x < 20^{2.5} - 2$ $x < 1786.85$ So $x \leq 1786$ or $x < 1787$.	Solves equation to find $x = 1786.85$.	Correct solution for x as a whole number.	

2019 Question 1a-b.

(a)(i)	$3x^2 - 7x - 6 = (3x + 2)(x - 3) = 0$ $x = \frac{-2}{3}, 3$ or equivalent.	Correct solutions.		
(a)(ii)	$5x^2 - 4x - 3 = 0$ $x = 1.27, -0.47$ or equivalent.	Correct solutions.		
(b)	$5.05 = 0.02t^2 - 0.6t + 9.18$ $0.02t^2 - 0.6t + 4.13 = 0$ $t = 10.7, 19.3$ Hence $t = 10.7$ months (cannot be 19.3)	Quadratic equation set equal to 0.	Correct answer.	

2019 Question 2a-c.

(a)(i)	$9^{0.5}a^1b^{-2}$ $= \frac{3a}{b^2}$	Correct answer.		
(a)(ii)	$\left(\frac{3b^4}{2a}\right)^2 = \frac{9b^8}{4a^2}$	Correct answer.		
(b)	$\frac{2c+1}{(c+3)(c-3)} + \frac{c-1}{(c-3)(c-1)}$ $= \frac{(2c+1)(c-1) + (c-2)(c+3)}{(c+3)(c-3)(c-1)}$ $= \frac{3c^2 - 7}{(c+3)(c-3)(c-1)}$	Cross-arrangement to a single fraction.	Final simplification.	
(c)	$fm - 2gm - 6gn + 3fn$ $= m(f - 2g) + 3n(f - 2g)$ $= (m + 3n)(f - 2g)$	Pairs factored.	Complete factorisation.	

2018 Question 1a-d.

(a)	$25^{\frac{1}{2}}(m^{16})^{\frac{1}{2}}$ $= 5m^8$	Correct answer.		
(b)	$\left(\frac{3a}{4}\right)^2 = \left(\frac{9a^2}{16}\right)$	Correct answer.		
(c)	$\frac{4(3c)}{3c} - \frac{b+8c}{3c} = \frac{4c-b}{3c}$	Fractions written with a common denominator.	Final simplification.	
(d)	$4bx + 2xy - 6ab - 3ay$ $= 2x(2b + y) - 3a(2b + y)$ $= (2x - 3a)(2b + y)$	Complete factorisation.		

2018 Question 3a-b.

(a)(i)	$12x^2 - 5x - 2 = 0$ $(4x + 1)(3x - 2) = 0$ $x = \frac{-1}{4}$ or $\frac{2}{3}$ or equivalent	Correct solutions.		
(a)(ii)	$x^2 + x - 3 = 0$ $x = 1.303, -2.303$	Correct solutions.		

2017 Question 1a-b.

ONE (a)(i)	$3(4x)^{-2} = \frac{3}{16x^2}$	Correct answer.		
(a)(ii)	$\left(\frac{16x^4}{x^6}\right)^{\frac{3}{2}} = (16x^{-2})^{\frac{3}{2}}$ $= \left(16^{\frac{3}{2}}x^{-3}\right)$ $= \frac{64}{x^3}$	Correct power of x or number.	Correct answer.	

2016 Question 1a-b.

ONE (a)	$\left(\frac{c^2}{3b}\right)^4 = \frac{c^8}{81b^4}$	Negative or fourth power correctly used.	Correct answer.	
(b)	$x^2 - 8x + 10 = (x - 4)^2 - 6$	Correct arrangement or p and q given. ($p = 4$ and $q = -6$ not required)		

2016 Question 3a.

THREE (a)	$12x^2 - x - 6 = (4x - 3)(3x + 2)$ $x = \frac{3}{4} \text{ or } -\frac{2}{3}$	Correct solutions.		
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2016 Question 3c.

(c)	$\frac{4x}{5} = \frac{y(x+3)}{2}$ $8x = 5y(x+3)$ $x(8-5y) = 15y$ $x = \frac{15y}{8-5y}$	Terms involving x collected to one side.	Correctly solved.	
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2015 Question 2a-c.

(a)	$\frac{(x+4)(2x-1)}{2(x^2-16)}$ $= \frac{(x+4)(2x-1)}{2(x-4)(x+4)}$ $= \frac{(2x-1)}{2(x-4)}$ Provided $x \neq \pm 4$	Factorised and simplified with one error.	Correctly simplified.	
(b)	$a^7 = \left(y^{\frac{3}{4}}\right)^7$ $= y^{\frac{21}{4}}$	Correct expression.		
(c)	Let $x = u^{\frac{1}{3}}$ $2x^2 + 7x - 4 = 0$ $(2x-1)(x+4) = 0$ $x = \frac{1}{2} \text{ or } x = -4$ $u^{\frac{1}{3}} = \frac{1}{2}$ so $u = \frac{1}{2^3} = \frac{1}{8}$ OR $u^{\frac{1}{3}} = -4$ so $u = (-4)^3 = -64$	CAO or rewrites as quadratic.	Solves for x .	Solves completely with both solutions.

2015 Question 3a-b.

(a)(i)	$\left(\frac{a^{10}}{4a^5}\right)^{-2} = \left(\frac{4a^5}{a^{10}}\right)^2$ $= \left(\frac{4}{a^5}\right)^2 = \frac{16}{a^{10}}$	Evidence of correctly simplifying the negative or index or square or numerator or denominator correct	Algebraic expression simplified.	
(ii)	$\sqrt[5]{\left(\frac{32}{x^5}\right)^3} = \left(\frac{32}{x^5}\right)^{\frac{3}{5}}$ $= \frac{32^{\frac{3}{5}}}{(x^5)^{\frac{3}{5}}} = \frac{(\sqrt[5]{32})^3}{x^3}$ $= \frac{8}{x^3}$	Numerator or denominator correct.	Algebraic expression simplified.	
(b)	$\frac{1}{t(t-1)} - \frac{t-1}{t(t-1)} - \frac{3t}{t(t-1)} = 0$ $\frac{1-t+1-3t}{t(t-1)} = 0$ $\frac{2-4t}{t(t-1)} = 0$ $\frac{2(1-2t)}{t(t-1)} = 0$ $t = \frac{1}{2}$	Partially solved by rewriting over correct common denominator.	Correctly solved.	

2014 Question 1a-b.

(a)(i)	$\frac{a^{12}}{125}$	Correct.		
(ii)	$0.5x^{1.5}$ or $0.5x^{\frac{3}{2}}$	Correct equivalent.		
(iii)	$\frac{2x^2}{3x^{-8}} = \frac{2}{3}(x^{10})$	Numerator or denominator correct.	Correct expression.	
(b)	Let the roots be n and $3n$. $3n^2 = 12$ $n^2 = 4$ $n = \pm 2$ $4n = \pm m$ $m = \pm 8$	Relationship shown.	Values of n found.	Values of m found.

2014 Question 1d.

(d)	$(5x^2 - 4)(2x^2 - 1)$ $x^2 = 0.8$ or 0.5 $x = \pm 0.894$ or ± 0.707	Expression factorised.	Solved for x^2 .	All solutions given.
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2013 Question 1a.

(a)(i)	$(2x - 5)(3x + 2)$ $6(x + \frac{2}{3})(x - \frac{5}{2})$	Correctly factorised. Correct decimal / rounding.		
(a)(ii)	$x = -\frac{2}{3}(-0.67)$ or $x = \frac{5}{2}(2.5)$	Complete correct solutions found. Any rounding / truncation. Consistency with a(i) but not trivial (coefficients of $x > 1$)		

2013 Question 1c.

(c)	$\frac{2(x+2)(x-2)}{(x+2)(x-4)}$ $= \frac{2(x-2)}{(x-4)}$ accept $\frac{(2x-4)}{(x-4)}$	Factorised. OR Factorised and simplified with one error in factorising.	Correctly simplified.	
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2013 Question 2a-b.

(a)(i)	$(3a - 5)(4a + 3)$ $a = \frac{5}{3}$ or $-\frac{3}{4}$	Correct factorising or solution.	Correct solution showing factorisation.	
(b)(i)	$\frac{3x+3-4x^2+8x}{(x-2)(x+1)}$ $= \frac{-4x^2+11x+3}{x^2-x-2}$	Correct single fraction	Correctly simplified. Accept with denominator in factorised form.	
(ii)	$\frac{(x+4)(x-2)}{(x+1)(x-2)} = 3$ $x+4 = 3x+3$ if $x \neq 2$ $2x = 1$ $x = 0.5$	Factorised and simplified CAO.	Solved for x .	Condition $x \neq 2$.

2013 Question 2a-d.

(a)	$64a^6/64a^{10}$ $= 1/a^4$ (or a^{-4})	Correct simplification.		
(b)(i)	$2x^{0.5}$ (or $2\sqrt{x}, 2x^{\frac{1}{2}}$	Correct simplification.		
(ii)	$2x^{0.5} \times 3x^{1.5} = 6x^2$	Correct simplification. Consistent with (b) (i).		
(c)	$6x^2 + 12x - 48 = 0$ $x^2 + 2x - 8 = 0$ $(x + 4)(x - 2) = 0$ $x = -4$ and $x = 2$	Correct equation = 0 OR CAO or guess and check. OR If answer correct & $x = -4$ is eliminated.	Equation solved showing equation.	
(d)	$x \log a = (x - 1) \log 5$ $= x \log 5 - \log 5$ $x(\log a - \log 5) = -\log 5$ $x = \frac{-\log 5}{\log a - \log 5}$		Expression written in log form and expanded.	Expression for x correct. OR equivalent.

2012 Question 1c.

(c)	$x^2 - 6x - 27 = 0$ $(x + 3)(x - 9) = 0$ $x = -3$ or $x = 9$ $3^n = -3$ no solution Only solution is $3^n = 9$ $\Rightarrow n = 2$	Quadratic equation in x formed and solved.	Expression given for 3^n .	Value of n found with algebraic evidence.
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2012 Question 2a-d.

TWO (a)(i)	$(5x+1)(x-2)$ OR $5(x-2)(x+0.2)$	Factorise the expression.		
(ii)	$x = -\frac{1}{5}$ or 2 or equivalent.	Equation solved giving TWO solutions. Accept in fractional form. Consistent with 2a(i) but not trivial.		
(b)	$x^2 + 5x + 2 = 3x + 6$ $x^2 + 2x - 4 = 0$ $x = 1.236, -3.236$	Expanded and simplified to a quadratic equation = 0. CRO Truncate / rounding ok – min 1dp.	Equation solved giving TWO correct solutions. Truncate / rounding ok – min 1dp.	
(c)	$\frac{(x-3)(x-2)}{(x+3)(x-2)} = 4$ $\frac{(x-3)}{(x+3)} = 4$ $3x = -15$ $x = -5$	Two solutions $x = -5, x = 2$, with comments about incorrect factorisation (or the correct factorisation). OR An answer to the question with both values substituted showing these solutions do not =4.	Correct solution of $x=-5$ only with one of the two aspects of the incorrect solution discussed.	Correct solution and i) A comment about incorrect factorisation. ii) $x = 2$ gives an invalid solution as it results in dividing by 0 or back substitution shows $x = 2$ does not satisfy the equation. BOTH required.
(d)	$\frac{(x+3)(x-2)}{6x^2+4x+c} = \frac{x+3}{2(3x+8)}$ Multiply numerator and denominator of RHS by $(x-2)$ $\Rightarrow 2(x-2)(3x+8)$ $= 6x^2 + 4x - 32$ Therefore $c = -32$	Factorising, and recognising the need to multiply by $(x-2)$ to equate denominators. Or cross multiplication and expanding and simplifying correctly.	Solving.	

2012 Question 3a.

THREE (a)(i)	$8x^{13}$	Correct.		
(ii)	$4x^{\frac{1}{3}}$	Correct (accept 0.3 as power).		
(iii)	$\sqrt{\frac{4x^{\frac{1}{3}}}{x^{-\frac{1}{2}}}}$ $= 2x^{\frac{5}{12}}$	Consistent with 3a(ii) .	Correct. Or equivalent.	