Algebra Collated Past Papers - Algebraic Manipulation

2023 Question 1a-b.

(a)   Simplify each expression, leaving your answer with positive indices.

(i) $\sqrt[5]{\dfrac{4(2n)^3}{n^8}}$

(ii) $\left(\dfrac{n^3}{16n^6}\right)^{-0.5}$

(b) $\dfrac{20x^2-22x+6}{5x^2-3x}$ can be written in the form $A+\dfrac{B}{x}$, where A and B are integers.

Find the values of A and B.

2023 Question 3a.

(a)   (i)   Solve the following equation:

$5x^2+13x=6$

(ii)   Simplify:

$\dfrac{3x^2-10x+8}{9x^2-16}$

2023 Question 3c.

(c) $\dfrac{x^2+2x+k}{x^2+7x+10}=\dfrac{x-3}{x+2}$ where k is a constant.

Work out the value of k.

2022 Question 1a-b.

(a)   Solve $\dfrac{2x-3}{x+4}-3=0$.

(b)   (i)   Factorise completely $6x^3y-15x^2\sqrt{y}$.

(ii)   Simplify fully $\dfrac{6x^2-x-12}{3x^2-5x-12}$.

2022 Question 2c.

(c)   Solve $\sqrt{2x+3}=3x$.

2022 Question 3a.

(a) (i) Simplify fully $\sqrt{49y^{36}}$.

2021 Question 1a-b.

(a) Simplify each expression, leaving your answer with positive indices.

(i) $\dfrac{(3y)^4}{3y^{-1}}$

(ii) $\sqrt[3]{8y^{27}}$

(b) A quadratic equation has solutions of $x = -\dfrac{2}{3}$ and $x = 4$.

Find the original equation, giving your answer in the form of $ax^2 + bx + c = 0$, where a, b, and c are whole numbers.

2021 Question 2a-b.

(a) Simplify: $\dfrac{x^2 - x - 12}{4x + 12}$

(b) Write $\dfrac{5x}{x-3} - \dfrac{x-4}{x+2}$ as a single fraction in its simplest form.

2020 Question 1a-c.

(a) Factorise $6x^2 + 13x - 15$.

(b) A function is defined as $f(x) = x^2 + 10x + 22$.

Express $f(x)$ in completed square form, i.e. $f(x) = (x + a)^2 + b$, where a and b are integers.

(c) In the 16th century, mathematicians were developing a formula to solve any cubic equation. They used expressions in the form of $y = x^3 - 12Px + R$, where P and R are positive constants.

(i) The graph of $y = x^3 - 12Px + R$, for some values of P and R, passes through the point A (4,40) and is sketched below.
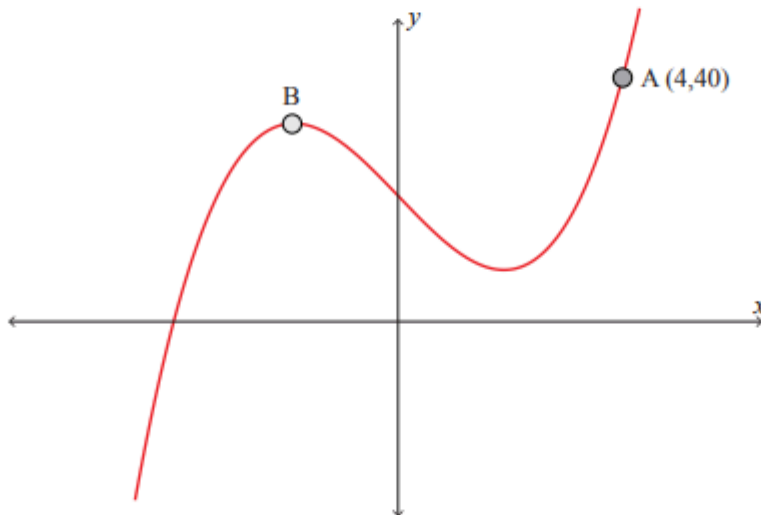
(ii) At point B it is true that $3x^2 - 12P = 0$.

Find an expression for P in terms of R.

Using algebra, show that $x = -2P^{0.5}$ at B.

2020 Question 2c.

(c)   Solve the equation: $\dfrac{5x+4}{x+4}-\dfrac{3x-4}{2x+1}=2$.

2020 Question 3b.

(b)   Consider the function $W=(x+2)^{\frac{2}{5}}$, where x is a **whole number**.

(i)   Make x the subject of the formula $W=(x+2)^{\frac{2}{5}}$.

(ii)   For what values of x will the function have values less than 20?

2019 Question 1a-b.

(a)   Solve each of the following equations:

(i) $3x^2-6=7x$

(ii) $\dfrac{3}{x^2}+\dfrac{4}{x}=5$

(b)   A drug is used to reduce the level of cholesterol in the blood. For a daily dose of the drug, the cholesterol level C in the blood t months after taking the first dose may be modelled by the function

$$C=0.02t^2-0.6t+k,$$

where k is the initial cholesterol level and the function is valid for the first 15 months only.

A person with an initial cholesterol level of 9.18 is given the drug.

How long will it take the person's cholesterol level to reduce to 5.05?

2019 Question 2a-c.

(a)   Simplify fully, leaving your answers with positive indices:

(i) $(9a^2b^{-4})^{0.5}$

(ii) $\left(\dfrac{2a}{3b^4}\right)^{-2}$

(b)   Write $\dfrac{2c+1}{c^2-9}+\dfrac{c-2}{c^2-4c+3}$ as a single fraction in its simplest form.

(c)   Factorise fully $fm-6gn+3fn-2gm$.

2018 Question 1a-d.

(a) Simplify fully $\left(25m^{16}\right)^{\frac{1}{2}}$

(b) Simplify fully $\left(\dfrac{4}{3a}\right)^{-2}$, leaving your answer with a positive index.

(c) Write $4 - \dfrac{b+8c}{3c}$ as a single fraction in its simplest form.

(d) Factorise fully $4bx + 2xy - 6ab - 3ay$

2018 Question 3a-b.

(a) Solve each of the following equations for x:

    (i) $12x^2 - 5x = 2$ (ii) $x+1-\dfrac{3}{x}=0$

(b) Show that the graph of the function $y = 2x^2 - 5x + 6$ does not cross the x-axis.

     You must use algebra to support your explanation.

2017 Question 1a-b.

(a) Simplify the following, leaving your answer with positive indices:

    (i) $3(4x)^{-2}$ (ii) $\left(\dfrac{16x^4}{x^6}\right)^{\frac{3}{2}}$

(b) Fully simplify the expression $\dfrac{2x^2 - 50}{9x^2 - 39x - 30}$.

2016 Question 1a-b.

(a) Simplify $\left(\dfrac{3b}{c^2}\right)^{-4}$ leaving your answer with positive indices.

(b) Write $x^2 - 8x + 10$ in the form $(x - p)^2 + q$.

2016 Question 3a.

(a) Where would the graph of $y = 12x^2 - x - 6$ cut the x-axis?

2016 Question 3c.

(c) Rearrange the following formula to make x the subject: $\dfrac{4x}{5} = \dfrac{y(x+3)}{2}$.

2015 Question 2a-c.

(a) Simplify $\dfrac{2x^2 + 7x - 4}{2x^2 - 32}$ (b) If $a = y^{\frac{3}{4}}$, find an expression for a^7 in terms of y.

(c) Solve the equation $2u^{\frac{2}{3}} + 7u^{\frac{1}{3}} = 4$

2015 Question 3a-b.

(a) Simplify, giving your answer with positive exponents:

(i) $\left(\dfrac{a^{10}}{4a^5}\right)^{-2}$

(ii) $\sqrt[5]{\left(\dfrac{32}{x^5}\right)^3}$

(b) Solve the following equation for t:

$$\frac{1}{t(t-1)} - \frac{1}{t} = \frac{3}{t-1}$$

2014 Question 1a-b.

(a) Simplify:

(i) $\left(\dfrac{5}{a^4}\right)^{-3}$

(ii) $\left(0.25x^3\right)^{\frac{1}{2}}$

(iii) $\dfrac{\left(8x^6\right)^{\frac{1}{3}}}{3\left(x^{-2}\right)^4}$

(b) One root of the equation $x^2 + mx + 12 = 0$ is three times the other.

Find the values of m.

2014 Question 1d.

(d) Solve $10x^4 - 13x^2 + 4 = 0$

You must show algebraic working.

2014 Question 2a-b.

(a) Factorise and solve $12a^2 - 11a - 15 = 0$

(b) (i) Write as a single fraction $\dfrac{3}{x-2} - \dfrac{4x}{x+1}$

(ii) Solve the equation $\dfrac{x^2 + 2x - 8}{x^2 - x - 2} = 3$

You must show algebraic working.

2013 Question 1a.

   (a) (i)   Factorise $6x^2 - 11x - 10$ (ii)   Solve $6x^2 - 11x - 10 = 0$

2013 Question 1c.

   (c) Simplify fully $\dfrac{2x^2 - 8}{x^2 - 2x - 8}$

2013 Question 2a-d.

   (a) Simplify $\dfrac{(4a^2)^3}{(8a^5)^2}$ (b) Simplify:

                                         (i) $(16x^2)^{\frac{1}{4}}$ (ii) $(16x^2)^{\frac{1}{4}} \times (9x^3)^{\frac{1}{4}}$

   (c) Lara says that she is thinking of a number. She:

        •   squares the number,

        •   multiplies the answer by 6,

        •   adds 12 times the number she was first thinking of,

        •   subtracts 48.

      Her answer is 0.

      What numbers could she be thinking of?

   (d) Rearrange the formula $a^x = 5^{(x-1)}$ to make x the subject.

2012 Question 1c.

   (c) Solve $9^n - (6 \times 3^n) - 27 = 0$ and explain why it has only one real solution.

      Hint: let $3^n = x$

2012 Question 2a-d.

   (a) (i)   Factorise $5x^2 - 9x - 2$

        (ii)   Solve $5x^2 - 9x - 2 = 0$

   (b) Solve $\dfrac{x^2 + 5x + 2}{x + 2} = 3$

      Show algebraic working.

2012 Question 2a-d cont.

(c) Mark solves the equation $\dfrac{x^2-5x+6}{x^2+x-6}=4$

His working is shown below.

$x^2 - 5x + 6 = 4x^2 + 4x - 24$ _____

$3x^2 + 9x - 30 = 0$ _____

$3(x^2 + 3x - 10) = 0$ _____

$3(x + 6)(x - 2) = 0$ _____

$x = -6$ or $x = 2$ _____

Is Mark's answer correct?

Fully justify your answer.

(d) Find the value of c if $\dfrac{x^2+x-6}{6x^2+4x+c}=\dfrac{x+3}{2(3x+8)}$

2012 Question 3a.

(a) Simplify

(i) $(x^5)^2(2x)^3$

(ii) $\left(8x^{\frac{1}{2}}\right)^{\frac{2}{3}}$

(iii) $\sqrt{\dfrac{\left(8x^{\frac{1}{2}}\right)^{\frac{2}{3}}}{x^{\frac{-1}{2}}}}$