Algebra Collated Past Papers - Quadratics - Answers

2023 Question 1d.

(d)	$x^2 - k(2x+29) + 32k = 0$ $x^2 - 2kx + 3k = 0$ using $b^2 - 4ac = 0$ $4k^2 - 12k = 0$ $k = 0$ and $k = 3$ So, $k = 3$, so quadratic can be written as $y = \frac{x^2}{6} + 16$ When $x = 0$, $y = 16$	• First step in solving simultaneous equations: substitution for x or y, or equivalent, to give an equation in **one variable** only OR y-intercept correc	• Setting discriminant = 0 for relevant equation (allow minor error) OR Calculus used correctly to obtain both k and y-intercept correctly	T: k correct. TT: both k and y-intercept correct.

2023 Question 3b.

(c)	$\frac{x^2+2x+k}{(x+5)(x+2)} = \frac{x-3}{x+2}$ $x^2+2x+k = (x-3)(x+5)$ $x^2+2x+k = x^2+2x-15$ Therefore, $k = -15$ Or equivalent approach.	• Makes progress towards solution by eliminating denominators, or equivalent.	• Value found.	

2022 Question 2a-b.

TWO (a)	$(x - \frac{1}{3})(x + \frac{2}{7})$ $= (3x - 1)(7x + 2)$ $= 21x^2 - x - 2$ a = 21, b = –1, c = –2	Correct values of a, b, and c.		
(b)(i) (ii)	$(-12)^2 - 4(2)(7) = 88$ So $(-12)^2 - 4(2)(k)$ [= 0] 8k = 144 k = 18 accept use of inequality	Correct discriminant OR substitution made (line 1)	 Correct value of k.	

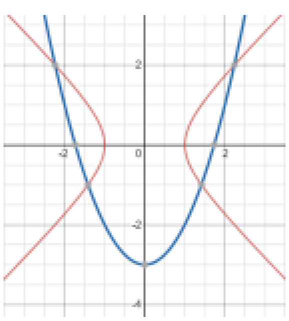
2022 Question 2d.

(d)(i)	$fx^2 + gx + h = hx^2 + gx + f$ $(f-h)x^2 + (h-f) = 0$ $(f-h)(x^2-1) = 0$ $x^2 = 1$ $x = 1$ or $x = -1$ Accept $\pm\sqrt{\dfrac{-(h-f)}{(f-h)}}$ or equivalent.		Correct working to obtain one solution only.	Both correct solutions obtained.

(ii)	Roots of $Q(x)$ are $x = \dfrac{-g \pm \sqrt{g^2-4fh}}{2f}$ Roots of $Q^*(x)$ are $x = \dfrac{-g \pm \sqrt{g^2-4hf}}{2h}$ If $A = \dfrac{-g-\sqrt{g^2-4fh}}{2f}$, then one of the roots of $Q^*(x)$ will be $kA = k\left(\dfrac{-g-\sqrt{g^2-4hf}}{2h}\right) = \dfrac{-g-\sqrt{g^2-4fh}}{2h}$ so $k = \dfrac{f}{h}$ OR If roots of $Q(x)$ are A and B, $AB = \dfrac{h}{f}$ If roots of $Q^*(x)$ are kA and kB, **(kA)(kB)** = $\dfrac{f}{h}$ So it follows that: $k^2AB = k^2\dfrac{h}{f} = \dfrac{f}{h}$ and $k^2 = \dfrac{f^2}{h^2}$ and $k = (\pm)\dfrac{f}{h}$ OR If roots of $Q(x)$ are A and B, $A+B = \dfrac{-g}{f}$ If roots of $Q^*(x)$ are kA and kB, **kA + kB** = $\dfrac{-g}{f}$ So it follows that: $k(A+B) = k\left(\dfrac{-g}{f}\right) = \dfrac{-g}{h}k$ and $k = \dfrac{f}{h}$	Correct expressions for all 4 roots obtained (may be combined) OR $AB = \dfrac{h}{f}$ OR $A + B = \dfrac{-g}{f}$	Correct **expression** involving k as function of f,g and / or h OR Finds $k = \dfrac{h}{f}$ [this results from saying the root of $Q(x)$ is kA and that of $Q^*(x)$ is A]	Finds $k = \dfrac{f}{h}$

2021 Question 1c-d.

(c)	$x^2 - 3kx + 2k^2 = 0$ $(x - k)(x - 2k) = 0$ So solutions are k and 2k, [hence one is twice the other]. OR $x^2 - 3kx + 2k^2 = 0$ $\Delta = (-3k) - 4(1)(2k^2) = k^2$ Hence, $x = \dfrac{-(-3k) \pm \sqrt{k^2}}{2}$ $= \dfrac{3k \pm k}{2}$ $= \dfrac{4k}{2}$ or $\dfrac{2k}{2}$ i.e. one root is $2k$ and the other is k and hence one is twice the other.	Equation factorised. OR Discriminant evaluated.	Correct conclusion with valid working.	

(d)	Curve Two: $y = (x - 1)(x + 1) - 2$ So $y = x^2 - 3$ Curve One: $x^2 = y^2 + 1$, so substitute into Curve Two to give: $y = y^2 + 1 - 3 = y^2 - 2$ $y^2 - y - 2 = 0$ $(y - 2)(y + 1) = 0$ Either $y = 2$ so $x^2 = 5$ and $x = \pm\sqrt{5}$, so points of intersection are $(\pm\sqrt{5}, 2) = (\pm 2.24, 2)$ Or $y = -1$ so $x^2 = 2$ and $x = \pm\sqrt{2}$, so points of intersection are $(\pm\sqrt{2}, -1) = (\pm 1.41, -1)$. FYI: 	Correct expansion and simplification of Curve Two.	Forms one equation in one variable.	T1: Correct identification of two solutions only or of four x values. T2: Complete correct solution with correct mathematical statements.

2020 Question 2d.

(d)	$ax^2 + bx + c = dx^2 + ex + c$ $(a - d)x^2 + (b - e)x = 0$ $x[(a - d)x + (b - e)] = 0$ so $x = 0$ or $x = \frac{e-b}{a-d}$ One solution will always be on the y-axis, i.e. $x = 0$. The other is $\frac{e-b}{a-d}$. Hence, there will always be one solution, and there will always be a second as long as $a \neq d$ so that this second solution is defined and $b \neq e$, so that the second is distinct from the first. Accept alternative method: use of quadratic formula to derive the same results.	Sets up simultaneous equation.	Solves quadratic correctly but does not draw conclusions from the solutions.	T1: Correct working leading to one of the constraints, clearly expressed. T2: Correct working leading to both of the constraints, clearly expressed.

2019 Question 1c-e.

(c)	Discriminant $\Delta = b^2 - 4ac = 0$ $(m+1)^2 - 4(2m-1)(m-4) = 0$ $7m^2 - 38m + 15 = 0$ $(7m-3)(m-5) = 0$ $m = 5$ ($\frac{3}{7}$ eliminated as $m - 4$ must be at least 0 since the expression is a perfect square)	Sets up discriminant equal to 0.	Sets up quadratic equation equal to 0.	Correct solution incorporating clear explanation of the rejection of $m = \frac{3}{7}$.
(d)	$p^2x^2 + 4px - 12 = (px+6)(px-2) = 0$ $x = \frac{-6}{p}, \frac{2}{p}$ Hence $\frac{2}{p} - \frac{-6}{p} = \frac{8}{p}$.	Factorises correctly or finds the difference between roots consistently.	The correct roots are found.	Finds the difference between the correct roots.
(e)	$y = x^2 - bx - ax + ab - c^2$ $= x^2 - (a+b)\,x + (ab - c^2)$ $\Delta = (-(a+b))^2 - 4 \times 1 \times (ab - c^2)$ $= (a^2 - 2ab + b^2) + 4c^2$ $= (a-b)^2 + 4c^2$ As $(a-b)^2 \geq 0$ and $c^2 > 0$ then $\Delta > 0$ and hence there are two real distinct roots (and two distinct points where the graph crosses the x–axis). Graphical argument: "Since $f(x) = (x-a)(x-b)$ is a positive parabola which clearly has 2 roots and its graph crosses the x-axis at 2 distinct points, the function $g(x) = f(x) - c^2$, which must be lower since c^2 must be positive, must also have 2 distinct roots (which would be further apart than a and b)."	Function is set up so that the discriminant can be found.	Finds discriminant in factored form.	Explains why the discriminant is greater than 0 and makes a correct conclusion. OR graphical argument is fully described.

2019 Question 2e.

(e)	Roots are $\frac{-p+\sqrt{p^2-4q}}{2}$ and $\frac{-p-\sqrt{p^2-4q}}{2}$ $-p+\sqrt{p^2-4q} = n(-p-\sqrt{p^2-4q})$ $\sqrt{p^2-4q} = \frac{p(1-n)}{(1+n)}$ $qn^2 + (2q-p^2)n + q = 0$	One root n times the other.	Successfully squares both sides.	Finds equation with correct algebraic working.

2018 Question 1f.

(f)	$3x^2 - 36xy + xy - 12y^2 - 2x^2 + 32xy - xy + 16y^2$ $= x^2 - 4xy + 4y^2$ $= (x - 2y)^2$ $a = x$, $b = -2y$ (or vice versa)	Correct expansion.	Correct simplification.	Square completed and a and b identified correctly.

2018 Question 3c-d.

(c)	$3(3)^2 + k(3) - 12 = 0$ $27 + 3k - 12 = 0$ $k = -5$ $3x^2 - 5x - 12 = 0$ $(3x + 4)(x - 3) = 0$ $x = \frac{-4}{3}$ OR $(3x + e)(x - 3) = 3x^2 + kx - 12$ Equating coefficients for constant term, $e = 4$ Hence other root is $-\frac{4}{3}$	Forms a factor of $(x - 3)$ and uses it in a valid way. OR Finds $k = -5$.	Correct answer.	
(d)	For equal roots $\Delta = (2(k + 1))^2 - 4(-k^2 - 2k - 5) = 0$ $\Rightarrow 4k^2 + 8k + 4 + 4k^2 + 8k + 20 = 0$ $8k^2 + 16k + 24 = 0$ $k^2 + 2k + 3 = 0$ and for this quadratic $\Delta = 2^2 - 4 \times 1 \times 3 = -8$ (or -512 etc.) < 0, or $(k + 1)^2 = -2$ etc. So there are no real solutions and hence no values of k for which the original equation has equal roots.	Correct discriminant substitution.	Simplified quadratic set equal to 0.	Correct conclusion. 1t: Concluded that there are no real roots with little or no working shown. 2t: Shows $\Delta < 0$ from quadratic, or draws graph to conclude that no real roots exist.

2017 Question 3a-e.

THREE (a)	$4\left(x+\frac{1}{2}\right)\left(x-\frac{5}{2}\right)=0$ $(2x+1)(2x-5)=0$ $4x^2-8x-5=0$ $b=-8$	Finds b.		
(b)	$6x^2-mx+3=0$ One unique real root means $\Delta=0$ $(-m)^2-4\times6\times3=0$ $m^2=72$ $m=\pm8.485$	Correct substitution into discriminant and set to 0.	Correct values for m.	
(c)	There are no real roots so: $\Delta<0$ $(12)^2-4k(5k)<0$ $144-20k^2<0$ $k^2>\frac{144}{20}$ Either $k>2.68$ or $k<-2.68$ Graph is always above the x-axis so $k>0$ and it follows $k>2.68$	Recognising graph is above x-axis with discriminant $\Delta=b^2\text{-}4ac<0$	Finds -2.68 and 2.68	Correct range of values for k.
(d)	$\frac{9}{(x-3)(x+3)}+\frac{3}{2(x+3)}$ $=\frac{18+3(x-3)}{2(x-3)(x+3)}=\frac{3x+9}{2(x-3)(x+3)}$ $=\frac{3(x+3)}{2(x-3)(x+3)}$ $=\frac{3}{2(x-3)}$ $x\neq3$ or -3	Lowest common denominator found.	Correct answer. Restriction on values of x need not be given.	
(e)	$2^{mx-3}=8^{x^2}=\left(2^3\right)^{x^2}=2^{3x^2}$ $mx-3=3x^2$ $3x^2-mx+3=0$ has exactly one root. $\Delta=0$ $m^2-4\times3\times3=0$ $m^2=36$ $m=6$ or -6	Changes bases to 2.	Substitutes and makes discriminant equal to 0.	Correctly solved.

2016 Question 1c-e.

(c)(i)	When $x^2 + x - 56 = 0$ $(x+8)(x-7) = 0$ $x = -8$ or 7. When $4x^2 + x - 14 = 0$ $(4x-7)(x+2) = 0$ $x = \frac{7}{4}$ or -2 So the solutions of the first quadratic are four times those of the second.	Both equations factorised correctly if solution is incorrect.	Both quadratics solved and relationship stated.	
(c)(ii)	Solutions of $dx^2 + ex + f = 0$ are $\frac{-e \pm \sqrt{e^2 - 4df}}{2d}$ and those of $x^2 + ex + df = 0$ are $\frac{-e \pm \sqrt{e^2 - 4df}}{2}$ So the solutions of the second quadratic are *d* times those of the first.	One set of solutions found.	All solutions found.	Devised a strategy and developed a chain of logical reasoning to solve the problem.
(d)	$\left(x+\frac{1}{2}\right)\left(x-\frac{2}{3}\right)=0$ $(2x+1)(3x-2) = 0$ $6x^2 - x - 2 = 0$ So $a = 6$, $b = -1$ and $c = -2$ or any other correct values of a, b, and c.	Quadratic found with correct values of *a*, *b*, and *c*.	Correct values of *a*, *b* and c stated.	

(e)	To have rational roots, the discriminant is • ≥ 0 (accept > 0 at achieved and merit) • a perfect square Hence $16k^2 - 4 \times 2(2k^2 + 3k - 11) \geq 0$ $-24k + 88 \geq 0$, $k \leq \frac{88}{24}$ or $\frac{11}{3}$ Cases, *k* integer $k = 0$ not possible as told *k* positive $k = 1$, $\Delta = 64$ which is a square $k = 2$, $\Delta = 40$ which is not a square $k = 3$, $\Delta = 16$ which is a square $k \geq 4$ not possible as Δ negative So only possible values are $k = 1$ or 3.	Values substituted into discriminant.	$k \leq \frac{11}{3}$ or unsimplified. OR One value for *k* found with reason. OR Discriminant used correctly with one restriction on *k* given.	Both values of *k* found with logical chain of reasoning.

2016 Question 2a.

TWO (a)	$x^2 - 10x - 3 = 0$ discriminant $= 100 + 4 \times 3$ $= 112$	Correct discriminant found.		

2015 Question 3c-d.

(c)	Never touch the x-axis means $\Delta < 0$ $(3k-1)^2 - 4(2k+10) < 0$ $9k^2 - 6k + 1 - 8k - 40 < 0$ $9k^2 - 14k - 39 < 0$ If $9k^2 - 14k - 39 = 0$ Then $(9k+13)(k-3) = 0$ and $k = 3$ or $-\frac{13}{9}(-1.44)$ So if the graph does not cut the x-axis, then $-\frac{13}{9} < k < 3$	$\Delta < 0$	Correct solutions for k.	Problem solved with correct inequality.

(d)	If both roots real, so $\Delta > 0$ ie $[-(m+2)]^2 - 4m \times 2 > 0$ So $m^2 - 4m + 4 > 0$ i.e. $(m-2)^2 > 0$ So m can be any real number but $m \neq 2$, as any number squared except zero is always positive. Using the quadratic formula or otherwise, the roots are $\frac{m+2 \pm \sqrt{(m-2)^2}}{2m}$ $= \frac{m+2 \pm (m-2)}{2m}$ $x_1 = \frac{2m}{2m} = 1$ provided $m \neq 0$ or $x_2 = \frac{4}{2m} = \frac{2}{m}$ provided $m \neq 0$ So to fill all conditions including both roots are positive real, we have $m > 0$ m≠2 with roots 1 and $\frac{2}{m}$.	$\Delta > 0$	$(m-2)^2 > 0$ and $m \neq 2$ OR BOTH roots found.	Problem solved correctly.

2013 Question 1b.

(b)	$b^2 - 4ac = 0$ $16m = 64$ $m = 4$	Perfect square. $4(x-1)(x-1) = 0$ or equivalent $x = 1$ $m = 4$	Recognising the discriminant must = 0. OR CRO.	Calculating the value of m.	

2013 Question 1d-e.

(d)	$a^2 - 3a - 4 = 0$ $(a-4)(a+1) = 0$ $a = 4$ or $a = -1$ $\sqrt{(x+2)} = 4$ or $\sqrt{(x+2)} = -1$ (not a solution) $x = 14$	$((x+2) - 4)^2$ $= (3\sqrt{(x+2)})^2$ $x^2 - 4x + 4 = 9(x+2)$ $x^2 - 13x - 14 = 0$ $(x+1)(x-14) = 0$ $x = -1$ and $x = 14$	Equation rearranged and factorised OR Solved using either method. (RANW= n)	Solved for x. $x = 14$ and -1 but not disregarding $x = 0$.	Recognition that $x = -1$ is not a solution.
(e)(i) (ii)	$x^2 - mx + nx - mn = 2(x^2 - nx + mx - mn)$ $x^2 + 3mx - 3nx - mn = 0$ $x^2 + (3m - 3n)x - mn = 0$ $x = \frac{-3(m-n) \pm \sqrt{9(m-n)^2 + 4mn}}{2}$ $= \frac{-3(m-n) \pm \sqrt{9m^2 - 14mn + 9n^2}}{2}$ Hence $9m^2 - 14mn + 9n^2 > 0$ OR $9(m-n)^2 + 4mn > 0$		Cross multiplication and collection of like terms. Mei for one incorrect simplification step.	Correct substitution into the quadratic formula, not necessarily simplified. OR No roots given but correct inequality in (ii).	Roots and inequality found.

2013 Question 2e.

(e)	$(3x + n)(x - 2) = 0$ $3x^2 + (n - 6)x - 2n = 0$ $n - 6 = 4$ $n = 10$ $(3x + n) = 0$ root is $-n / 3 = -10 / 3$ $k = 2n = 20$	Establishing the relationship. OR CRO of k and other root.	One value for n or k or the other root found with algebraic working.	Solutions found for k and the other root with algebraic working.

2013 Question 3c.

(c)	$9x^2 + 6x + 8 = 0$ $b^2 - 4ac = -252$ therefore no real roots. Graph of the parabola does not cut the x-axis.	A squared term can never be negative hence there is no solution therefore the graphs do not intersect each other.	Quadratic expression rearranged=0 OR Explanation of no x intercepts because the discriminant is less than zero, without -252. OR A squared term can never be negative hence there is no solution.	Discriminant found and therefore no real roots but no x axis analysis. OR Quadratic expression rearranged = 0. AND Explanation of no x intercepts because the discriminant is less than zero, without -252.	Discriminant calculated and explanation of no x intercepts given. OR Full explanation of the two graphs not intersecting.

2012 Question 3b-c.

(b)(i)	$2x^2 - 3x + 8x - 12 = 13$ $2x^2 + 5x - 25 = 0$ $a = 2$, $b = 5$ and $c = -25$ $x = 2.5$ or $x = -5$	Expanding and simplifying to $= 0$. Incorrect simplification, then correct use of quadratic formula giving two solutions. CRO.	Solution including values for a, b, c.	
(ii)	$2x^2 + 5x - 12 - k = 0$ For one solution $b^2 - 4ac = 0$ $25 + 8(12+k) = 0$ $k = -15.125$	Knowledge of statement $b^2 - 4ac = 0$. Incorrect substitution into $b^2 - 4ac$.	Correct substitution into $b^2 - 4ac$.	Value of k calculated.
(c)	$x^2 + 5x - 1 - dx^2 - d = 0$ $x^2(1 - d) + 5x - (1 + d) = 0$ To have solutions $25 + 4(1 - d)(1 + d) > 0$ $25 + 4 - 4d^2 > 0$ $4d^2 < 29$ $-2.69 < d < 2.69$	Expansion and simplified equation –collecting coefficients (line 2) .	Correct substitution into the discriminant of $b^2 - 4ac > 0$. Including $>$ or $\geq$.	Range for d calculated. Do not penalise for using ≥ 0.