Algebra Collated Past Papers - Quadratics

2023 Question 1d.

(d) $2y = 2x + 29$ is a tangent to the quadratic $x^2 - 2ky + 32k = 0$, where k is a non-zero constant.

Find the value of k and determine where the quadratic crosses the y-axis.

2023 Question 3b.

(b) Find the range of values of p for which the graph $f(x) = 2x^2 + 8x + p$ does not cross the x-axis.

2022 Question 2a-b.

(a) A quadratic equation, $ax^2 + bx + c = 0$, has solutions of $\frac{1}{3}$ and $\frac{-2}{7}$.

Find the values of the integers a, b, and c.

(b) (i) What is the discriminant of the equation $2x^2 - 12x + 7 = 0$?

(ii) Suppose $y = 2x^2 - 12x + k$, where k is a constant.

For what value of k will the equation $y = 0$ have exactly one solution?

2022 Question 2d.

(d) Suppose $Q(x) = fx^2 + gx + h$, where f, g, and h are constants. The "reciprocal polynomial" of $Q(x)$ is defined as $Q^*(x) = hx^2 + gx + f$, where the coefficients are in the reverse order.

(i) Find the solutions of the equation $Q(x) = Q^*(x)$.

(ii) Suppose that $Q(x) = 0$ has 2 different roots, A and B.

The roots of $Q^*(x) = 0$ are multiples of A and of B, i.e. the roots are kA and kB for some constant k.

Find an expression for k in terms of f, g, and / or h.

2021 Question 1c.

(c) Consider a quadratic equation in the form $x^2 - 3kx + 2k^2 = 0$, where k is a non-zero constant.

Show that one solution is twice the other solution.

2021 Question 1d.

(d) Consider the following two curves:

$x^2 = y^2 + 1$ and $y = (x - 1)(x + 1) - 2$

Find the co-ordinates of each intersection point of the two curves.

2020 Question 2d.

(d) Consider two parabolas:

- Parabola One given by $y = ax^2 + bx + c$ and
- Parabola Two given by $y = dx^2 + ex + c$, where a, b, c, d, and e are constants.

Use algebra to determine the restrictions on the values of a, b, c, d, and e that would ensure that the parabolas meet at two distinct points.

2019 Question 1c-e.

(c) The polynomial $p(x) = (2m - 1)x^2 + (m + 1)x + (m - 4)$ can be written as a **perfect square**.

Find the value(s) of m.

(d) By factorising, find an expression in terms of p for the difference between the roots of the equation $(px)^2 + 4px - 12 = 0$.

(e) Use algebra to show that the graph of the function $y = (x - a)(x - b) - c^2$, where $c \neq 0$, crosses the x-axis at two distinct points.

2019 Question 2e.

(e) One root of the equation $x^2 + px + q = 0$ is n times the other, where $n \neq 0$.

Show that $qn^2 + (2q - p^2)n + q = 0$.

2018 Question 1f.

(f) $(3x + y)(x - 12y) - (2x + y)(x - 16y)$ can be written in the form $(a + b)^2$.

Find expressions for a and b in terms of x or y.

2018 Question 3c-d.

(c) The equation $3x^2 + kx - 12 = 0$ has two real solutions.

If one of the solutions is $x = 3$, find the other solution.

(d) Show that the roots of the equation $x^2 + 2(k + 1)x - (k^2 + 2k + 5) = 0$, where k is a constant, can never be equal for any real number k.

2017 Question 3a-e.

(a) The quadratic equation $4x^2 + bx - 5 = 0$ has solutions $-\frac{1}{2}$ and $\frac{5}{2}$.

Find the value of b.

(b) For what value(s) of m does the equation $6x^2 - mx = -3$ have two equal roots?

(c) Find the value(s) for k for which the expression $kx^2 - 12x + 5k$ is always greater than zero.

(d) Write $\frac{9}{x^2-9} + \frac{3}{2x+6}$ as a single fraction in its simplest form.

(e) Find the value(s) of m for which the equation $2^{mx-3} = 8^{x^2}$ has exactly one solution.

2016 Question 1c-e.

(c) (i) Show that the solutions of the equation $x^2 + x - 56 = 0$ are four times the solutions of the equation $4x^2 + x - 14 = 0$.

(ii) Find the relationship between the solutions of the equation $dx^2 + ex + f = 0$ and the solutions of the equation $x^2 + ex + df = 0$, where d, e, and f are real numbers.

(d) A quadratic equation of the form $ax^2 + bx + c = 0$ has solutions $-\frac{1}{2}$ and $\frac{2}{3}$.

Find a possible set of values for a, b, and c.

(e) Find positive integer value(s) for k so that the quadratic equation $2x^2 + 4kx + (2k^2 + 3k - 11) = 0$ has **real rational** solutions.

Justify your answer.

2016 Question 2a.

(a) Find the discriminant of the quadratic equation $x^2 = 10x + 3$.

2015 Question 3c-d.

(c) For what value(s) of k does the graph of the quadratic function

$$y = x^2 + (3k - 1)x + (2k + 10)$$

never touch the x-axis?

(d) The quadratic equation

$$mx^2 - (m + 2)x + 2 = 0$$

has two positive real roots.

Find the possible value(s) of m, and the roots of the equation.

2014 Question 1c.

(c) The equation $3x^2 - nx + 5 = 0$ has two distinct roots.

Find the values of n.

2013 Question 1b.

(b) Find the value of m so that only one value of x satisfies the equation:

$$4x^2 - 8x + m = 0$$

2013 Question 1d-e.

(d) The equation $(x+2) - 3\sqrt{(x+2)} - 4 = 0$ has only one real solution.

Find the value of x.

(Hint: Let $a = \sqrt{(x+2)}$)

(e) (i) Find expressions, in terms of m and n, for the roots of the equation:

$$\frac{x-m}{x-n} = \frac{2(x+m)}{x+n}$$

(ii) Give an inequality, in terms of m and n, so that the equation has two distinct roots.

2013 Question 2e.

(e) The equation $3x^2 + 4x - k = 0$ has two distinct real roots.

If 2 is a root of this equation, find the value of k and the second root.

2013 Question 3c.

(c) Explain why the equation $(3x + 1)^2 = -7$ does not have any real solutions, and explain what this means graphically.

2012 Question 3b-c.

(b) (i) Mark is solving $(2x - 3)(x + 4) = 13$ by using the quadratic formula

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Give the values of a, b and c and hence solve the equation.

(ii) The equation $(2x - 3)(x + 4) = k$ has only one real solution.

Find the value of k.

(c) Find the possible values of d if real solutions exist for $x^2 + 5x - 1 - d(x^2 + 1) = 0$.