Algebra Collated Past Papers - Logarithms - Answers

2023 Question 1c.

(c)	Initially $t = 0$ $900 = 40 + ke^0$ $k = 860$ $450 = 40 + 860e^{-0.5t}$ $860e^{-0.5t} = 410$ $e^{-0.5t} = 0.477$ Take log of both sides: $\ln\left(e^{-0.5t}\right) = \ln 0.477$ $-0.5t = \dfrac{\ln 0.477}{\ln e}$ $t = 1.48$ years	• t = 0 substituted.	• k correctly found using $t = 0$.	• Correctly solved.

2023 Question 2a.

TWO (a)(i)	$3m + 1 = 2^4$ $3m = 15,\ m = 5$	• Correct solution.		
(ii)	$\log_x 64 = 2$ $x^2 = 64$ $x = \pm 8$ As base cannot be negative, $x = 8$ OR $x^6 = 64^3$ $x = \sqrt[6]{262\,144}$ $x = \pm 8$ As base cannot be negative, $x = 8$	• Written in an index form. OR $x = 8$ obtained with no consideration of $x = -8$.	• Correct answer with justification or evidence of negative value being disregarded.	

2023 Question 2b-d.

(b)	$\frac{5^{7x+6}}{5^{-2x}}=\left(5^3\right)^p$ $5^{7x+6-(-2x)}=5^{3p}$ $9x+6=3p$ $p=3x+2$ OR $p=\log_{125}\left(\frac{5^{7x+6}}{25^{-x}}\right)$ $=\log_{125}\left(\frac{5^{7x+6}}{5^{-2x}}\right)$ $=\log_{125}\left(5^{9x+6}\right)$ $=\log_5\left(\frac{5^{9x+6}}{3}\right)$ $=3x+2$	• Conversion to either 5^{3p} or 5^{-2x}. OR $x=\frac{p-2}{3}$ OR Log expression up to line 1, which is only one possible log approach.	• Correct answer (simplification not required)	
(c)	$6+\log_b\left(b^{-3}\right)+\log_b\left(b^{\frac{1}{2}}\right)=6-3\log(b)+\frac{1}{2}\log_b(b)$ $=6-3+\frac{1}{2}$ $=3\frac{1}{2}$	• Combine logs into 1 log term, e.g. $\log_b(b-2.5)$.	• Rewriting both log terms bringing down the power.	• Correct value, even if the candidate goes direct to the numerical values
(d)	$4^x-10=3\times 4^x$ $4^{2x}-3\times 4^x-10=0$ Let $u=4^x$ $u^2-3u-10=0$ $(u+2)(u-5)=0$ $4^x=-2$ *or* $4^x=5$ Negative value not valid so: $\log 4^x=\log 5$ $x=\frac{\log 5}{\log 4}$ $x=1.16$ Accept $\log_4(5)$	• Obtains 4^{2x} OR 16^x OR $(4^x)^2$) OR $3G4^x$	• Solved for 'u'.	• Correct value.

2022 Question 3a-c.

(ii)	$x\log(2)=\log(2022)$ $x=10.98$ Accept $\log_2(2022)$	Correct solution.		
(b)	$\log(3a)+2\log\left(\frac{a}{6}\right)$ $=\log(3a)+\log\left(\left(\frac{a}{6}\right)^2\right)$ $=\log\left(3a\left(\frac{a}{6}\right)^2\right)$ $=\log\left(\frac{a^3}{12}\right)$	Fraction not correctly simplified but otherwise correct.	Correct expression obtained with fraction correctly simplified.	
(c)(i)	$\log_2(x-\mathrm{a})-\log_2(x+\mathrm{a})=\mathrm{c}$ $\log_2\frac{x-\mathrm{a}}{x+\mathrm{a}}=\mathrm{c}$ $\frac{x-\mathrm{a}}{x+\mathrm{a}}=2^{\mathrm{c}}$ $x-\mathrm{a}=2^{\mathrm{c}}(x+\mathrm{a})=x2^{\mathrm{c}}+\mathrm{a}2^{\mathrm{c}}$ $x(1-2^{\mathrm{c}})=\mathrm{a}+\mathrm{a}2^{\mathrm{c}}=\mathrm{a}(1+2^{\mathrm{c}})$ so, $x=\mathrm{a}\frac{1+2^{\mathrm{c}}}{1-2^{\mathrm{c}}}$	Log expressions combined correctly.	Correct exponential equation obtained (line 3).	Correct mathematical statements lead to the required expression.
(ii)	Using the expression from (c) part (i) … Firstly, if x is not defined, there will be no solutions, so that means that $1-2^{\mathrm{c}}\neq 0$, so $2^{\mathrm{c}}\neq 1$, and $\mathrm{c}\neq 0$. Hence c cannot be zero. Secondly, if a = 0, then $x=0$, but then the logs will be undefined. Hence, a cannot be zero. [Although, in the original equation, if a = 0 **and** c = 0, any strictly positive x-value is a solution, but the expression for x is undefined] Thirdly, for the original equation to be defined, both $x-\mathrm{a}>0$ and $x+\mathrm{a}>0$ (accept one or the other, or both).		One constraint identified with reasoning.	Two constraints identified with reasoning.

2021 Question 2c.

(c)(i)	D = 11, so $\left(1+\frac{R}{100}\right)^{11}=2$ $\left(1+\frac{R}{100}\right)=\sqrt[11]{2}=1.065$ $R=6.5(\%)$	Sets up equation and uses the 11th root. OR CAO	Obtains correct solution.	
(c)(ii)	$\left(1+\frac{R}{100}\right)^{D}=2$ $\log\left[\left(1+\frac{R}{100}\right)^{D}\right]=\log 2$ $D\log\left(1+\frac{R}{100}\right)=\log 2$ $D=\frac{\log(2)}{\log\left(1+\frac{R}{100}\right)}$	Logs taken of both sides (one of lines 2 or 3 must be shown).	Given expression derived correctly (one of lines 2 or 3 must be shown).	
(c)(iii)	$\frac{72}{R}=\frac{\log(2)}{\log\left(1+\frac{R}{100}\right)}$ $72\log\left(1+\frac{R}{100}\right)=R\log(2)$ $\log\left(\left(1+\frac{R}{100}\right)^{72}\right)=\log\left(2^{R}\right)$ $\left(1+\frac{R}{100}\right)^{72}=2^{R}$ $2^{R}-\left(1+\frac{R}{100}\right)^{72}=0$	Sets equations equal.	Processes powers (line 3)	T1: Obtains given equation with incorrect mathematical statements. T2: Obtains final given equation with correct mathematical statements. *PL: It may be reqd to make T1 = (c)(ii), T2=(c)(iii)*

2020 Question 2a-b.

(a)	$\log\left(\frac{9y\times 4}{3y}\right)=\log(12)$	Correct solution.		
(b)(i)	$x^2=36$ $x=6$	Correct solution.		
(b)(ii)	$\log_5(2x^2)=4$ $2x^2=5^4=625$ $x^2=312.5$ $x=\pm 17.68$ (4sf) $x>0$, so only solution is $x=17.68$	Combines logs in a valid way.	Finds x.	T1: Correct solution with negative value rejected.

2020 Question 3a.

(a)	$3^{4x} = 30$ $4x\log 3 = \log 30$ $x = \frac{1}{4}\left(\frac{\log 30}{\log 3}\right) = 0.7740\ (4\text{sf})$	Expanded log form.	Correct solution.	

2019 Question 3a-d.

(a)	$\log_5(m) = 3$ $\Leftrightarrow\ 5^3 = m \Leftrightarrow m = 125$	Correct answer.		
(b)	$\log 6 - 2\log y$ $= \log\left(\frac{6}{y^2}\right)$	Correct answer.		
(c)	$\frac{3^{2n-1} + 3^{2n+1}}{3^{2n} - 3^{2n-4}} = \frac{3^{2n-4}(3^3 + 3^5)}{3^{2n-4}(3^4 - 1)}$ $= \frac{27+243}{81-1} = \frac{27}{8}$ or equivalent.	Correct answer only.	Finds common factor of numerator and denominator.	Correct answer.
(d)(i)	$3N_0 = N_0(1.053)^t$ $3 = (1.053)^t$ $\log(3) = t\log(1.053)$ $t = \frac{\log(3)}{\log(1.053)} = 21.27$ weeks	Taking log of both sides and t as a factor.	Correct answer.	
(d)(ii)	$\frac{4250}{2500} = (1 + \frac{r}{100})^{10}$ $1 + \frac{r}{100} = \sqrt[10]{1.7} = 1.0545$ Hence r = 5.45 and rate of change is 5.45%.	Sets up correct equation.	Finds $1 + \frac{r}{100}$.	Percentage rate of change found.

2018 Question 2a-c.

(a)	$x^5 = 243 \Rightarrow x = 3$ Accept $\sqrt[5]{243}$.	Correct solution.		
(b)	$4m - 1 = 3^2$ $4m = 10 \Rightarrow m = \frac{5}{2}$ or equivalent	Correct solution.		
(c)	$\frac{3^{4x+1}}{(3^2)^x} = (3^3)^{\frac{w}{3}}$ $\frac{3^{4x+1}}{3^{2x}} = 3^w \Rightarrow 4x+1-2x = w$ $2x = w-1$ $x = \frac{w-1}{2}$	Expressed as powers of 3.	Correct answer.	

2018 Question 2e.

(e)(i)	$25\,000 = 20\,000(1.0385)^n$ $\log 1.25 = n \log 1.0385$ $n = \frac{\log 1.25}{\log 1.0385} = 5.91$ Hence 6 years. Whole year required by question.	Taking *log* of both sides and n as a factor OR $n = 5.91$ years	Correct answer.	
(ii)	$2 = \left(1 + \frac{r}{100}\right)^{12}$ $1 + \frac{r}{100} = \sqrt[12]{2} = 1.0595$ Hence $r = 5.95$ and interest rate is 5.95%.	Sets up correct equation.	Finds $1 + \frac{r}{100}$.	Interest rate found.

2017 Question 2a-c.

TWO (a)	$2^{10} = x$ $x = 1024$	Correct answer.		
(b)	$x^2 = 49$ $x = 7$ or -7 As base cannot be negative, $x = 7$	Written in index form.	Correct answer with justification.	
(c)	$x = \log_{\sqrt{5}} \frac{1}{125}$ $(\sqrt{5})^x = \frac{1}{125}$ $\left(5^{\frac{1}{2}}\right)^x = 5^{-3}$ $\frac{x}{2} = {}^-3$ $x = {}^-6$	Written in index form.	Problem solved.	

2017 Question 2d-e.

(d)	Initially the computer is \$4699 so $A = 4699$ $1500 = 4699r^{4.25}$ $r^{4.25} = \frac{1500}{4699} = 0.3192$ $r = \sqrt[4.25]{0.3192}$ $r = 0.764$ Value after 6 years $= 4699 \times 0.764^6$ or \$937.26 or consistent with rounding.	Sets up equation with correct value for A.	Value of r found.	Problem solved.
(e)	$\left(\frac{px}{q} - 3\right)\log 81 = \log 243$ $\frac{px}{q} - 3 = \frac{\log 243}{\log 81} = \frac{5}{4}$ $px = \frac{17q}{4}$ $p = \frac{17q}{4x}$ or $\frac{4.25q}{x}$	Converts equation to exponent form.	Simplifies logs on right hand side.	Problem solved.

2016 Question 2b-d.

(b)	$\frac{4\log(u^3)}{\log u} = \frac{12\log u}{\log u}$ $= 12$	Power rule for logs in numerator used.	Correct answer.	
(c)	$P = 24\,990(0.88)^t$ $12\,495 = 24\,990(0.88)^t$ $0.5 = 0.88^t$ $t = \frac{\log 0.5}{\log 0.88}$ $= 5.422$ So it takes 5.422 years to halve in value.	CAO or equation set up and error made in solving.	Correct equation solved to find value of t. Accept $t = 6$ if working shown.	
(d)(i)	$x = 8^{\frac{2}{3}}$ $= 2^2 = 4$	Correct value found.		
(d)(ii)	If $u = \log_8 x$ Then $6u^2 + 2u - 4 = 0$ $u = \frac{2}{3}$ or -1 Either $\log_8 x = \frac{2}{3}$ "or" $\log_8 x = -1$ $x = 8^{\frac{2}{3}}$ or 8^{-1} so $x = 4$ or $\frac{1}{8}$	CAO. OR Quadratic formed..	Both values for u found.	Devised a strategy and developed a chain of logical reasoning to solve the problem. Both values of x found.

2016 Question 3b.

(b)	$\log_x 216 = 3$ $x^3 = 216$ so $x = 6$ or $\sqrt[3]{216}$	Correct answer.			

2016 Question 3d.

(d)	$9^{8n+6} = 27^{n^2-1} \times 3^{1-3n}$ $(3^2)^{8n+6} = 3^{3(n^2-1)} 3^{1-3n}$ $3^{16n+12} = 3^{3n^2-3+1-3n}$ $16n+12 = 3n^2 - 3 + 1 - 3n$ $3n^2 - 19n - 14 = 0$ $(3n+2)(n-7) = 0$ $n = -\frac{2}{3}$ or 7	Base changed to 3 in all terms.	Quadratic established.	Devised a strategy and developed a chain of logical reasoning to solve the problem. Correct values for n found.	

2015 Question 1a-c.

(a)(i)	$2^x = 1024$ $x = 10$	Equation solved.		
(a)(ii)	$3w + 1 = 4^2$ $3w = 15$ and $w = 5$	Equation solved.		
(a)(iii)	$x^2 = 4x + 12$ $x^2 - 4x - 12 = 0$ $(x - 6)(x + 2) = 0$ $x = 6$ or -2 But base must be positive $x = 6$ is the only solution	Sets up a quadratic equation.	Solved problem using quadratic, but gives both values.	Gives only valid solution with justification.
(b)	$2x \log a = (x+1) \log b$ $x(2\log a - \log b) = \log b$ $x = \frac{\log b}{2\log a - \log b}$	Takes logs of both sides and multiplies by indices.	Takes logs of both sides and rearranges.	Correctly solved.
(c)(i)	$P = A \times (1.03)^t$ Beginning of 1999, $t = 0$, $t = 16$, $P = 350\ 000$ $350\ 000 = A\,(1.03)^{16}$ $A = 218\ 108$ So price was $218 108 initially.	Sets up model correctly.	Answers question in context correctly.	
(c)(ii)	$218\,100\,(1.03)^t = 200\,000\,(1.035)^t$ $\frac{218\,100}{200\,000} = \left(\frac{1.035}{1.030}\right)^t$ $1.0905 = 1.004854369^t$ $t = \frac{\log 1.0905}{\log 1.004854369}$ $t = 17.89$ In 2016.	Set up correct equation.	Solved for t.	Correct year identified.

2014 Question 3a-c.

(a)(i)	$3^x = 81$ $x = 4$	Correctly solved.		
(ii)	$343 = x^3$ $x = 7$	Correctly solved.		
(b)	$\left(\frac{5}{4}\right)^x = 15$ $x\log 1.25 = \log 15$ $x = \frac{\log 15}{\log 1.25}$ $x = 12.14$	Expression simplified.	Written in log form.	x found.
(c)(i)	0.8 is the fraction of medication remaining after an hour.	Correct explanation.		
(ii)	$M = 224 \times 0.8^{t-0.5}$ $= 224 \times 0.8^{-0.5}$ $= 250.4$ mg	Statement with $t = 0$ and attempt to solve.	Correctly solved.	
(iii)	$49.6 = 250.4 \times 0.8^t$ $0.8^t = \frac{49.6}{250.4}$ $t\log 0.8 = \log\frac{49.6}{250.4}$ $t = 7.25$ hours	$M = 49.6$ recognised and attempt to solve.	Correctly used logs in attempt to solve	Correctly solved.

2013 Question 3a-b.

(a)(i)	$x^3 = 64$ $x = 4$	Correctly solved.		
(ii)	$\frac{2^{x+1}}{2^{3x}} = 32$ $= 2^5$ $2^{x+1-3x} = 2^5$ $1 - 2x = 5$ $x = -2$	CRO OR Whole equation in powers of two. OR Use of log, with exponents eliminated. eg: $(x + 1)\log 2$ $= \log 32 + x \log 8$	Correctly solved.	
(b)(i)	1800×0.6^n	Expression correct.		
(ii)	$100 > 1800 \times 0.6^n$ $0.6^n < \frac{1}{18}$ $n\log 0.6 < \log\frac{1}{18}$ $n > 5.658$ 6 years OR equivalent.	In/Equation rearranged in index form. OR CRO. OR Solved by guess and check. OR 5.7 years with working. OR Consistent use of 0.4^n give $n = 3.15$	Number of years found as a whole number ($n = 6$) Consistent use of 0.4^n using logs give $n = 4$ years (whole number).	

2013 Question 3a-b.

(d)	$x = (mx)^2$ $x(m^2x - 1) = 0$ therefore either $m^2x = 1$ $x = \frac{1}{m^2}$ if $x \neq 0$. OR $x = 0$ But log 0 is undefined Therefore $x = \frac{1}{m^2}$	Equation given in index form.	One solution found. $x = \frac{1}{m^2}$ OR Both solutions and $x = 0$ not disregarded.	Correctly solved with $x = 0$ disregarded. OR Use of log properties to solve completely. $X = 0$ still needs to be disregarded.

2012 Question 1a-b.

ONE (a)(i)	8	Complete correct solution found.		
(ii)	$x = 5^2$ $= 25$	Complete correct solution found.		
(b) (i)	Log equivalent formed $2250/2000 = 1.035^t$ $t = \log 1.125/\log 1.035$ $= 3.42$ years	Establishing log equation. Problem solved using substitution (at least 2 iterations).	Accept 3.42 or 4 (years) or any other rounding. CRO of 3.42 allowed. Do not accept 3 unless accompanied by algebraic working.	
(ii)	$2000(1.035)^{21} - 2000(1.035)^{18}$ $= 4118.863 - 3714.978$ $= 403.8846$ $= \$403.88$	Value after 18 or 21 years found.	Correct solution. CRO	
(iii)	The additional amount in the account between Tara's m^{th} and $(m + n)^{\text{th}}$ birthday. OR The difference in the amount from the m^{th} year to the $(m + n)^{\text{th}}$.			Correct statement.