Algebra Collated Past Papers - Logarithms

2023 Question 1c.

(c) The value, V, in dollars (\$), of a laptop can be modelled by $V = 40 + ke^{-0.5t}$, $t \geq 0$, where t is the time in years since the laptop was purchased. The original price of the laptop was \$900.

How long does it take for the laptop's value to be reduced to 50% of the original value?

2023 Question 2a-d.

(a) (i) Find m if $\log_2(3m + 1) = 4$.

(ii) Solve the following equation:

$3\log_x(64) = 6$.

(b) Find an expression for p in terms of x if $\dfrac{5^{7x+6}}{25^{-x}} = 125^p$.

(c) Find the value of $6 + \log_b\left(\dfrac{1}{b^3}\right) + \log_b\left(\sqrt{b}\right)$.

(d) Using an algebraic method, solve $4^x - \dfrac{10}{4^x} = 3$.

2022 Question 3a-c.

(a) (ii) Solve the equation $2^x = 2022$.

(b) Simplify the following expression fully, writing your answer as a single logarithm.

$$\log(3a) + 2\log\left(\frac{a}{6}\right)$$

(c) Consider the equation $\log_2(x - a) - \log_2(x + a) = c$, where a and c are constants.

(i) Show that when x is made the subject of this equation, $x = a\dfrac{1+2^c}{1-2^c}$.

Ensure that you use correct mathematical statements in your reasoning.

(ii) The equation $\log_2(x - a) - \log_2(x + a) = c$, is only possible to solve for some values of a and for some values of c.

Explaining your reasoning clearly, describe which values of a and of c will make the equation possible to solve.

You may find it useful to recall that, when x is made the subject of this equation, $x = a\dfrac{1+2^c}{1-2^c}$.

2021 Question 2c.

(c) Jessica is investigating a compounding investment. She wants to know how long it would take for an investment of \$1000 to double in value to \$2000. She forms the following equation:

$$2000 = 1000\left(1+\frac{R}{100}\right)^{D}$$

where R is the rate of return on the investment, as a percentage, and

D is the time that the investment would take to double in value, in years.

(i) If an investment takes 11 years to double in value, what is its rate of return?

(ii) By making D the subject of the expression: $2000 = 1000\left(1+\frac{R}{100}\right)^{D}$,

show that $D = \dfrac{\log(2)}{\log(1+\frac{R}{100})}$

In her research, Jessica comes across a simple but **approximate** rule for calculating D, the time that the investment would take to double in value. It is commonly called the 'Rule of 72', and it states that:

$$D = \frac{72}{R}$$

Jessica wonders how close the values of D from the 'Rule of 72' are to those calculated using the actual expression, which is:

$$D = \frac{\log(2)}{\log(1+\frac{R}{100})}$$

(iii) Show clearly that the value of R for which the 'Rule of 72' exactly calculates D, is the solution to the equation:

$$2^{R} - \left(1+\frac{R}{100}\right)^{72} = 0$$

You do not need to solve this equation.

2020 Question 2a-b.

(a) Write as a single logarithm in simplest form: $\log(9y) + \log(4) - \log(3y)$.

(b) Solve each of the following equations:

(i) $\log_x(36) = 2$.

(ii) $\log_5(x) + \log_5(2x) = 4$.

2020 Question 3a.

(a) Solve the equation $3^{4x} = 30$.

2019 Question 3a-d.

(a) Find the value of m if $\log_5 m - 3 = 0$

(b) Write as a single logarithm: $\log 6 - 2\log y$

(c) Fully simplify $\dfrac{3^{2n-1} + 3^{2n+1}}{3^{2n} - 9^{n-2}}$

(d) (i) The number of people N suffering from a contagious virus increases exponentially at a constant rate of 5.3% each week after the virus was initially diagnosed.

If N_0 is the number of people initially diagnosed with the virus, then t weeks after the virus was initially diagnosed, N can be modelled by the function $N = N_0\,(1.053)^t$.

How long will it take for the number of people diagnosed with the virus to be three times the number initially diagnosed?

(ii) The number of people N suffering from a different virus also increases exponentially at a constant rate of r % each week. 2500 people were initially diagnosed with this virus. After 10 weeks, the number of people suffering from this virus had increased to 4250.

Find r, assuming the form of model in part (i) still applies.

2018 Question 2a-c.

(a) Find x if $\log_x 243 = 5$

(b) Find m if $\log_3(4m - 1) = 2$

(c) Find an expression for x in terms of w if $\dfrac{3^{4x+1}}{9^x} = 27^{\frac{w}{3}}$

2018 Question 2e.

(e) Interest is compounded on a principal investment, \$$P$, **at the end of each year**.

If the total amount of the investment after n years is \$$A$ then $A = P\left(1 + \dfrac{r}{100}\right)^n$

where r % is the compound interest rate per year.

(i) Anushka invests \$20 000 at an interest rate of 3.85% (so $A = P(1.0385)^n$).

How many years will it take for her investment to be worth \$25 000?

(ii) Semisi invests his money at a different interest rate than Anushka's investment. His investment will double in value after twelve years.

What is the interest rate for Semisi's investment?

2017 Question 2a-b.

(a) Solve the following equation for x:

$\log_2 x = 10$

(b) Solve the following equation for x:

$\log_x 49 = 2$

Justify your answer.

2017 Question 2c-e.

(c) Find the value of $\log_{\sqrt{5}}\left(\frac{1}{125}\right)$.

(d) A computer depreciates continuously in value from \$4699 to \$1500 over a period of 4.25 years.

The value, \$$y$, of the computer t years after its value was \$4699 can be modelled by a function of the form

$y = Ar^t$, where r is a constant.

Find the computer's value after six years.

(e) Make p the subject of the formula:

$$81^{\left(\frac{px}{q}-3\right)} = 243$$

2016 Question 2b-d.

(b) Simplify $\frac{4\log\left(u^3\right)}{\log u}$.

(c) Marie buys a new car for \$24 990.

The car's value decreases continuously by 12% each year.

The value of the car, \$$P$, t years after she first bought it, can be modelled by a function of the form $P = A(r)^t$.

How long will it take for the value of the car to halve?

(d) (i) Solve the equation $\log_8 x = \frac{2}{3}$.

(ii) Solve the equation $6(\log_8 x)^2 + 2\log_8 x - 4 = 0$.

2016 Question 3b.

(b) For what value(s) of x does $\log_x(216) = 3$?

2016 Question 3d.

(d) Solve the equation $9^{8n+6} = 27^{n^2-1} \times 3^{1-3n}$.

2015 Question 1a-c.

(a) (i) Find the value of $\log_2 1024$.

(ii) Solve the equation $\log_4(3w + 1) = 2$.

2015 Question 1a-c cont.

(iii) Luka says that the equation $\log_x(4x + 12) = 2$ has only one solution.

Is he correct?

Find the solution(s), justifying your answer.

(b) Make x the subject of the equation $a^{2x} = b^{x+1}$.

(c) The market value of Sue's house has been increasing at a constant exponential rate of 3% per annum since she bought it sixteen years ago at the start of 1999. At the start of 2015 it was worth \$350 000.

(i) Assuming the exponential growth is of the form $y = A\,r^t$, what was the value of the house at the start of 1999 when she bought it?

(ii) A friend also bought a house at the start of 1999 that cost \$200 000.

Its market value also has been steadily increasing, but at a slightly higher exponential rate of 3.5%.

Its value, \$$y$, t years after the start of 1999, is given by the function

$$y = 200\,000 \times (1.035)^t$$

If the houses continue to keep increasing in value at the original rates, in which year will the two houses be worth the same amount?

2014 Question 3a-c cont.

(a) (i) Find the value of x if $x = \log_3 81$

(ii) Solve the equation $\log_x 343 = 3$

(b) Solve for x: $5^x \times 2^{-2x} = 15$

(c) Thirty minutes after a patient is administered his first dose of a medication, the amount of medication in his blood stream reaches 224 mg.

The amount of the medication in the blood stream decreases continuously by 20% each hour.

The amount of the medication M mg in the patient's blood stream after it is administered can be modelled by the function

$$M = 224 \times 0.8^{t-0.5}$$

where t is the time in hours since the drug was administered.

(i) Explain what the 0.8 represents in this function.

(ii) Find the amount of medication administered initially.

(iii) A second dose of the medication can be administered some time later, and again the amount of the medication in the patient's bloodstream from the second dose can be modelled by the same function as that for the first.

The total amount of the drug in the blood stream must never exceed 300 mg.

How long after administering the first dose can the second dose be administered?

2013 Question 3a-b.

(a) Solve the equations:

(i) $\log_x 64 = 3$ (ii) $\dfrac{3 \times 2^{x+1}}{8^x} = 96$

(b) At the beginning of his first year of study, Danny borrows $1 800 from his parents.

His parents reduce the amount he owes them by 40% at the end of that year, and each subsequent year he continues his studies.

Danny studies for several years and he does not make any repayments of the initial amount while he is studying.

(i) Write an expression for the amount $$A$ Danny owes his parents if he studies for n years.

(ii) Use your expression to find the minimum number of years for which Danny studies if he owes his parents less than $100 when he finishes studying.

2013 Question 3d.

(d) Solve the equation $\log x = 2\log(mx)$ for x in terms of m.

2012 Question 1a-b.

(a) Solve

(i) $\log x = 3\log 2$

(ii) $\log_5 x = 2$

(b) Tara's aunt invests $2000 for her when she is born.

The interest rate is 3.5% per year.

This rate does not change as long as the money stays invested.

The interest is added to the amount she has invested on her birthday each year.

The value of the investment after t years can be modelled by the equation

$$A = 2000 \times (1.035)^t$$

where the A is the value of the investment.

(i) How long would it take for the value of the investment to be $2250?

(ii) Tara reaches her 18th birthday.

Calculate how much **extra** the investment will be worth if she leaves the money invested for another 3 years beyond her 18th birthday.

(iii) Tara is calculating $2000 \times 1.035^m(1.035^n - 1)$

With reference to the investment, explain what Tara is calculating.