Forming Equations From Graphs

Parabolas

Parabolas are U-shaped curves with a single vertex. There are two main approaches to writing their equations:

1. **Using the Vertex:** If you know the vertex coordinates, (b, c), you can use the following equation:
   - y = A(x - b)^2 + c

     Here, A determines the "stretch" of the parabola. A positive A opens the parabola upwards, while a negative A opens it downwards.

2. **Using the X-Intercepts:** If you know the two x-intercepts of the parabola, b and c, you can use this equation:
   - y = A(x - b)(x - c)

     Substitute these x-intercept values into the equation and solve for A.

Steps to solve for the equation:

1. Choose the appropriate equation based on the information you have (vertex or x-intercepts).
2. Substitute known values (vertex coordinates or x-intercepts) into the equation.
3. Locate another point on the parabola and substitute its x and y values into the equation.
4. Solve the equation for A, the scale factor that determines the parabola's stretch.

Cubics

Cubics are S-shaped curves with a single point of inflection. Similar to parabolas, there are two ways to write their equations:

1. **Using the Point of Inflection:** If you know the coordinates of the point of inflection, (b, c), you can use this equation:
   - $y = A(x - b)^3 + c$

   A determines the overall shape and direction of the curve.

2. **Using the X-Intercepts:** If you know all three x-intercepts of the cubic function, b, c, and d, you can use this equation:
   - $y = A(x - b)(x - c)(x - d)$

Steps to solve for the equation:

1. Choose the appropriate equation based on the information you have (point of inflection or x-intercepts).
2. Substitute known values (point of inflection coordinates or x-intercepts) into the equation.
3. Locate another point on the cubic and substitute its x and y values into the equation.
4. Solve the equation for A, the scale factor that determines the overall shape of the curve.

Hyperbolas

Hyperbolas are curves with two branches approaching, but never touching, asymptotes. Here's the general form for a hyperbola:

- $f(x) = A(x - b) + c$

Steps to solve for the equation:

1. Locate the vertical asymptote ($x = b$) and the horizontal asymptote ($y = c$). These indicate the hyperbola's horizontal and vertical shifts.
2. Substitute the values of b and c from the asymptotes into the general equation.
3. Locate another known point on the hyperbola and substitute its x and y values into the equation.
4. Solve the equation for A, the scale factor that determines the stretch of the hyperbola's branches.

Exponential Functions

Exponential functions are characterized by their rapid growth or decay. They have a horizontal asymptote that the curve infinitely approaches.

Steps to solve for the equation:

1. Locate the horizontal asymptote (y = c). This represents the function's long-term behavior.

2. Identify another exact point on the curve. The x-coordinate of this point is the value of b.

3. Determine the vertical distance between the point and the horizontal asymptote. This distance represents the value of k. A positive k indicates an upward growth, while a negative k indicates a downward decay.

4. Substitute a known point (x, y) other than the one used for b into the equation:
    - $y = k * a^{(x - b)} + c$
5. Solve the equation for a, the base of the exponential term.